Assumption-lean weak limits and tests for two-stage adaptive experiments

International Seminar on Selective Inference
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Co-author

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RCT versus adaptive experiment

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Motivating example: new treatment development for lowering CVD risk.[1]

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RCT versus adaptive experiment

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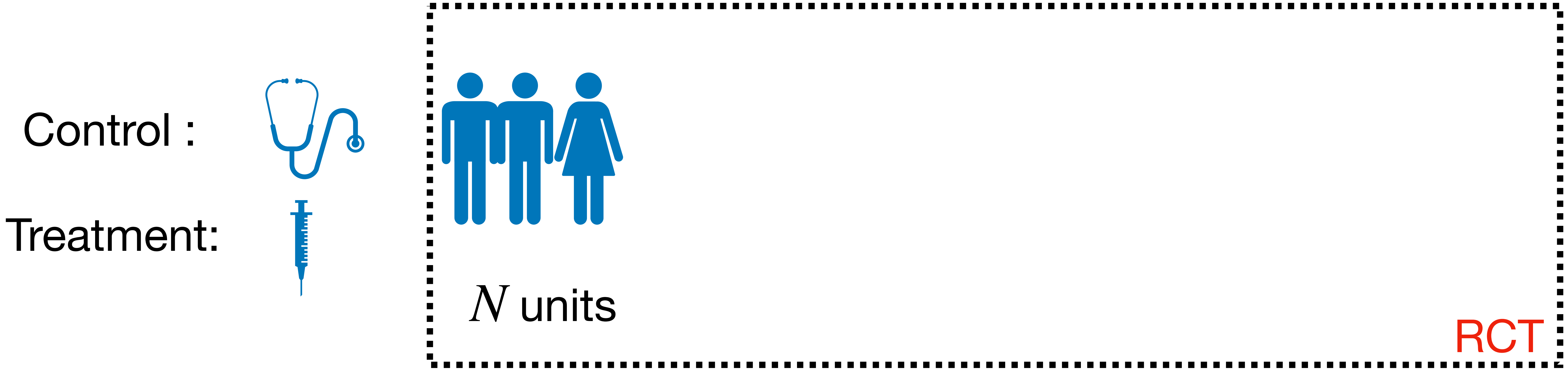

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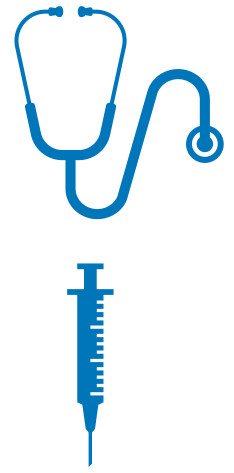

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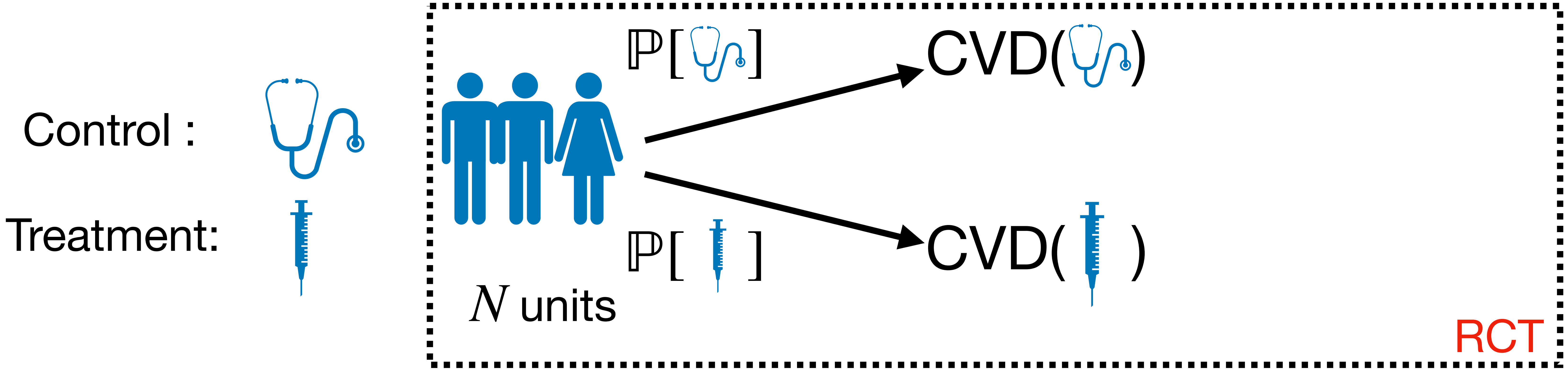

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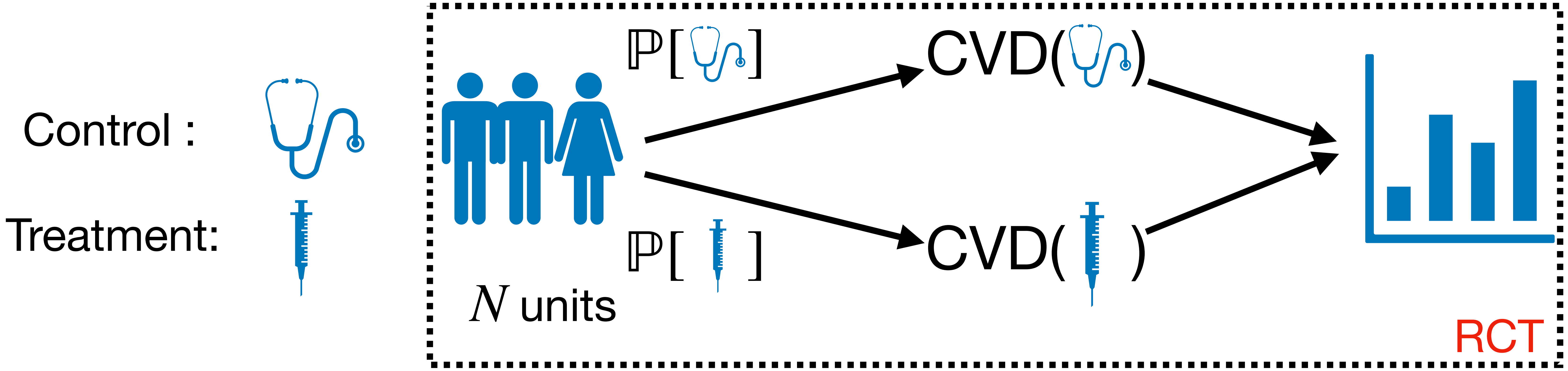

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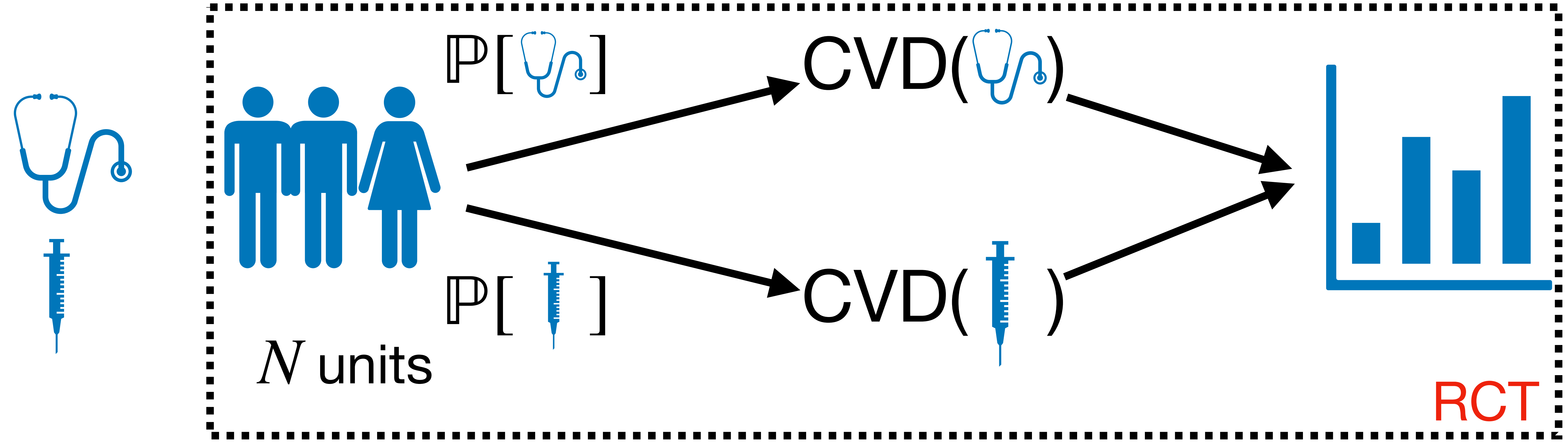

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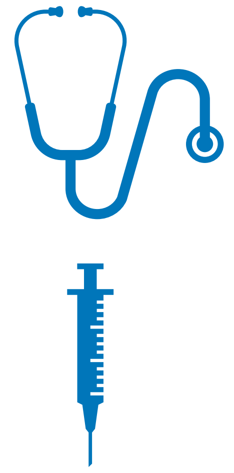

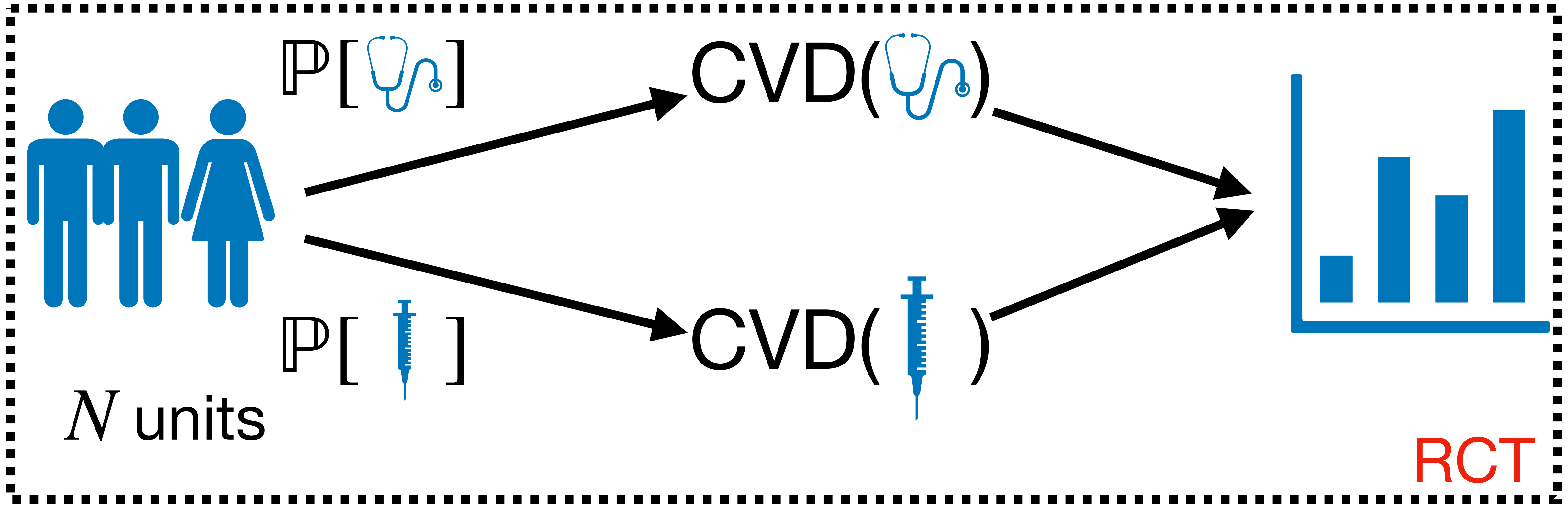

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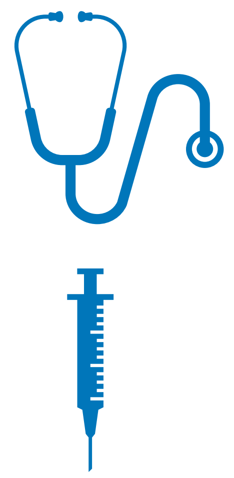

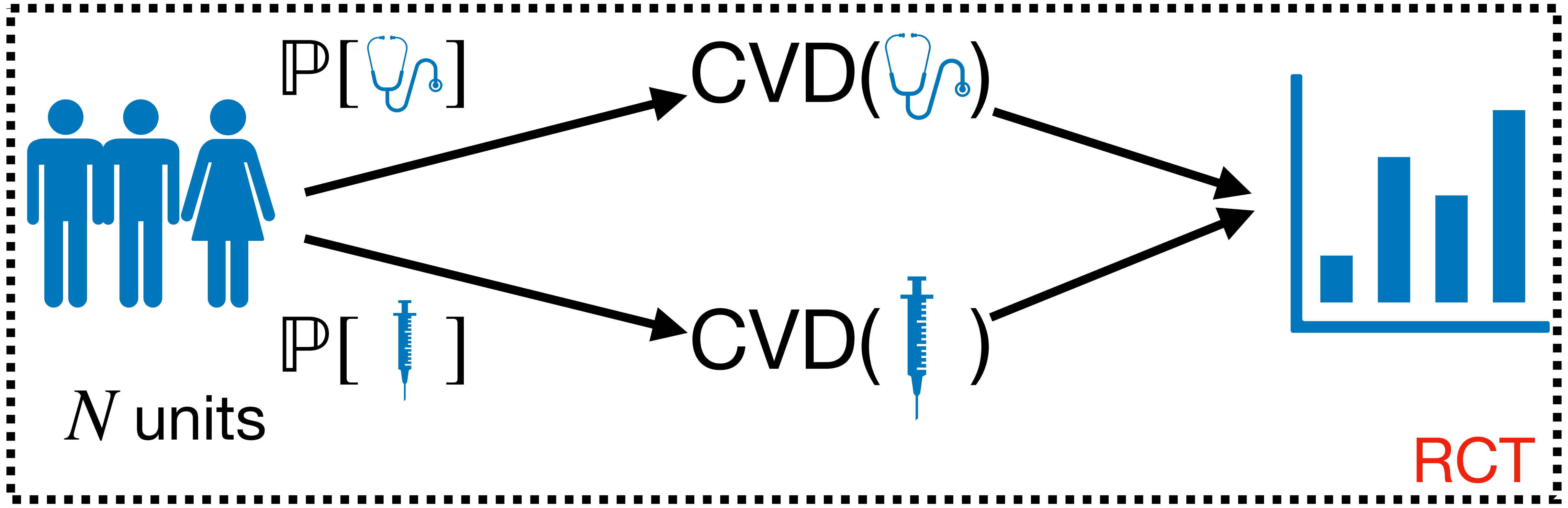

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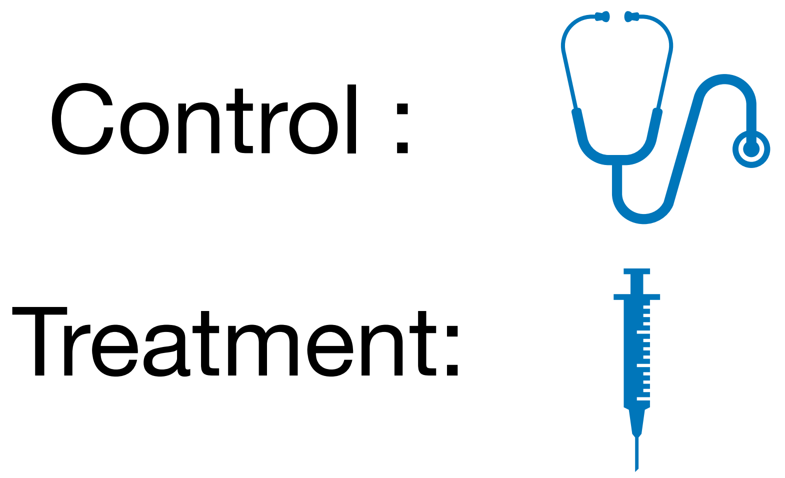

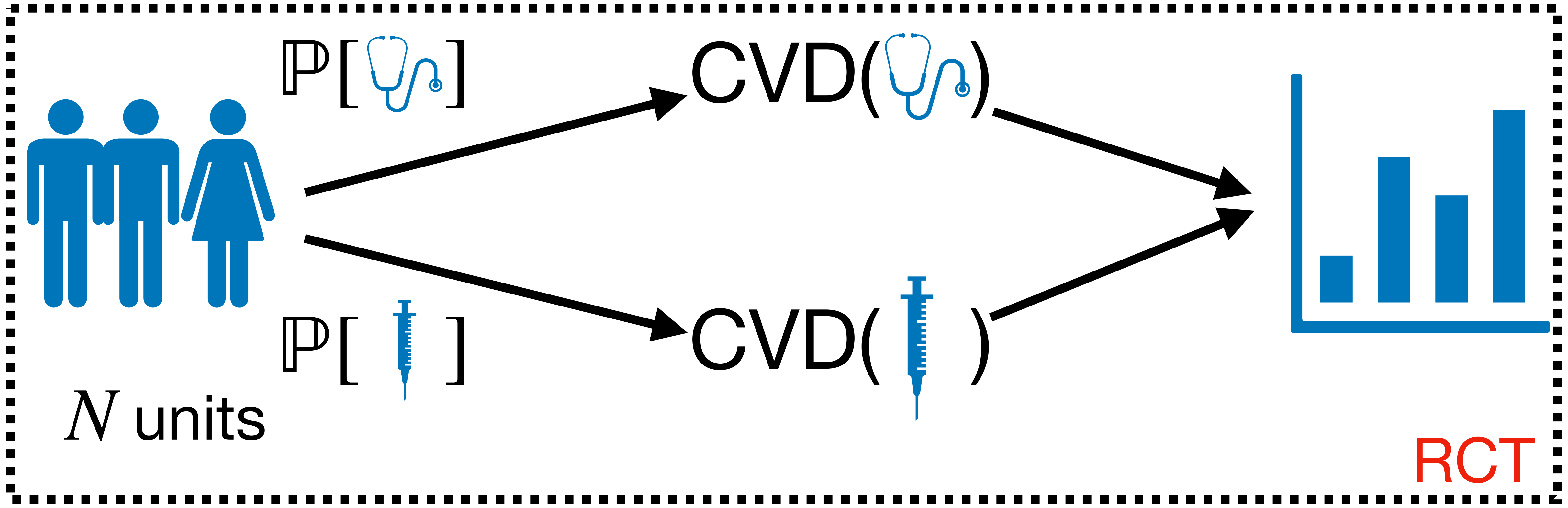

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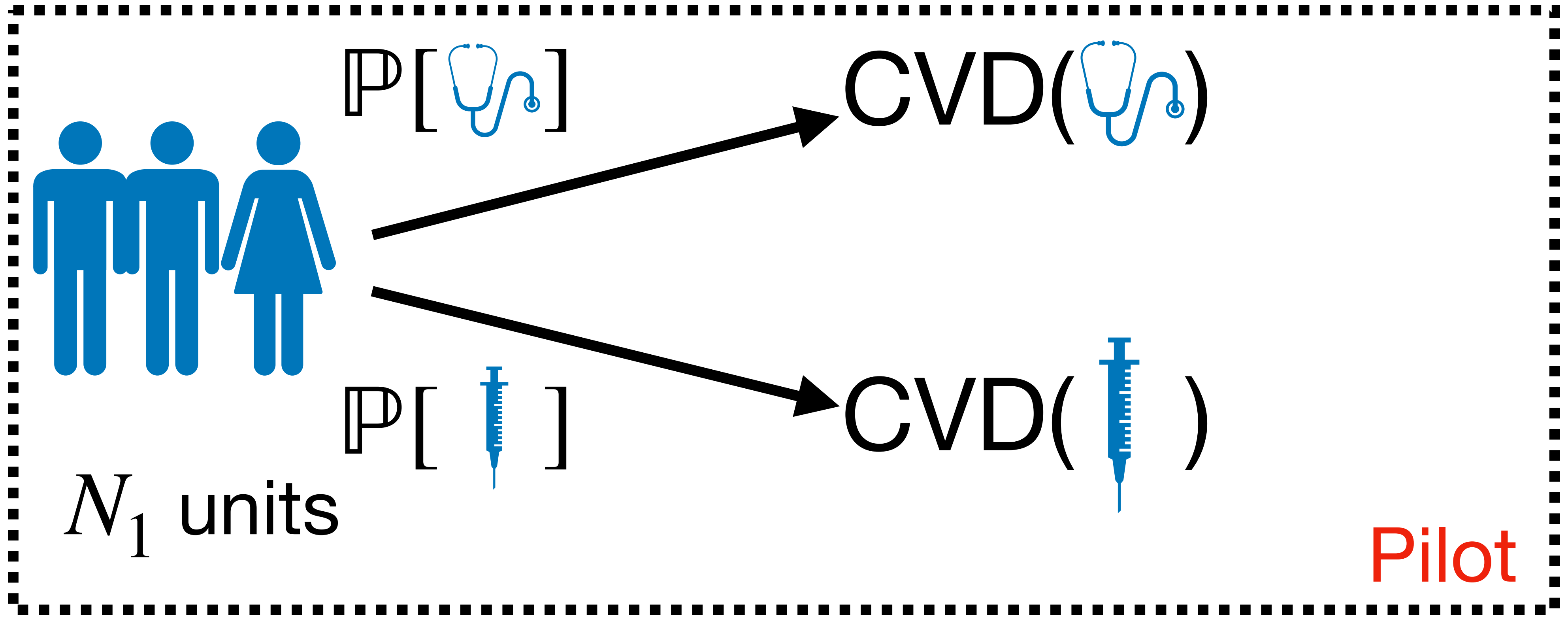

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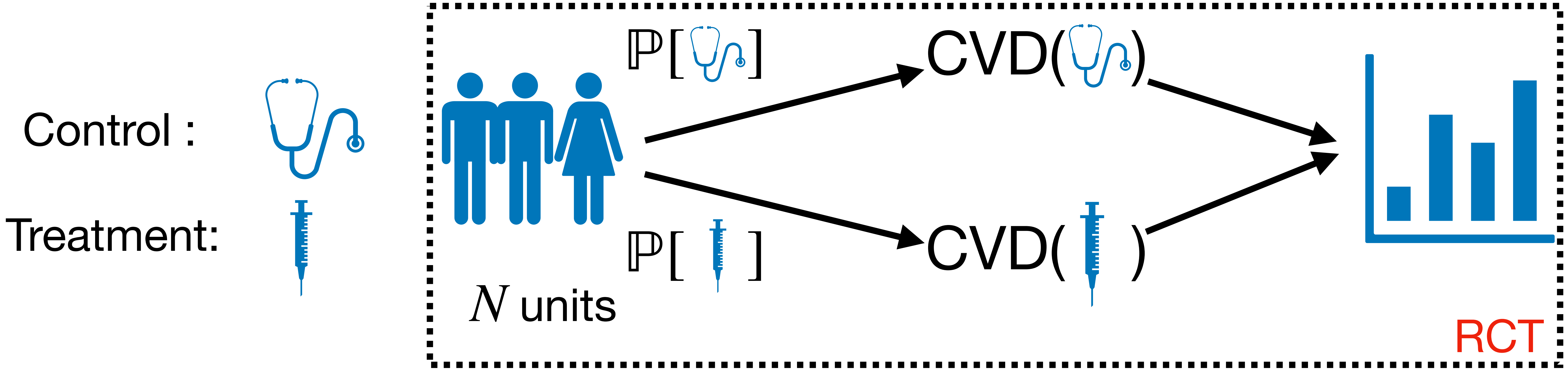

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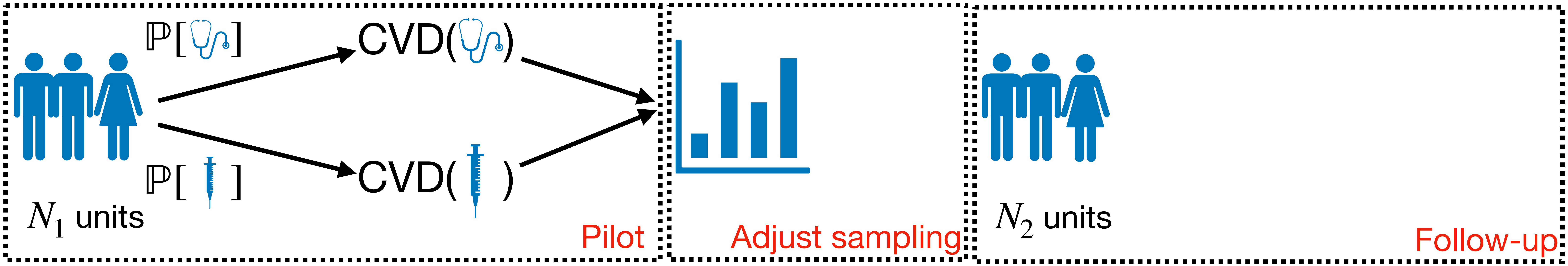

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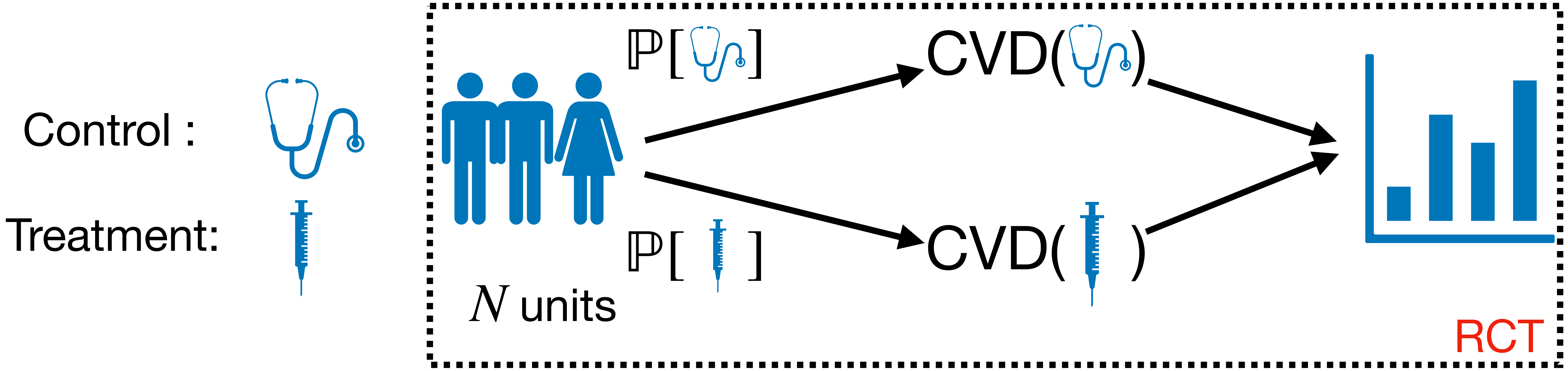

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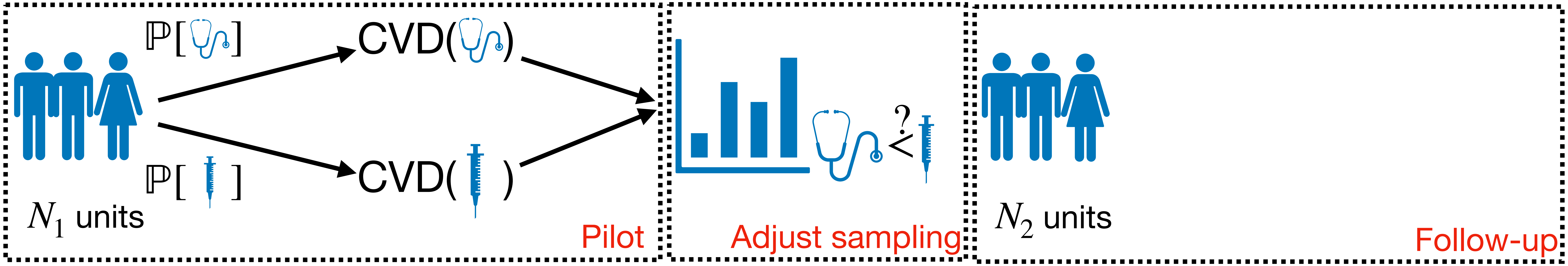

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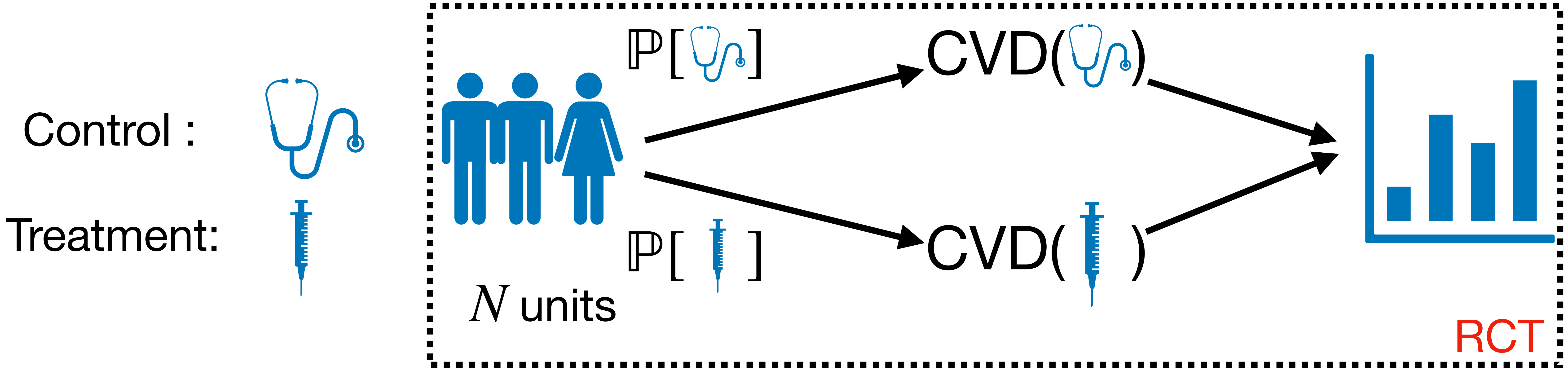

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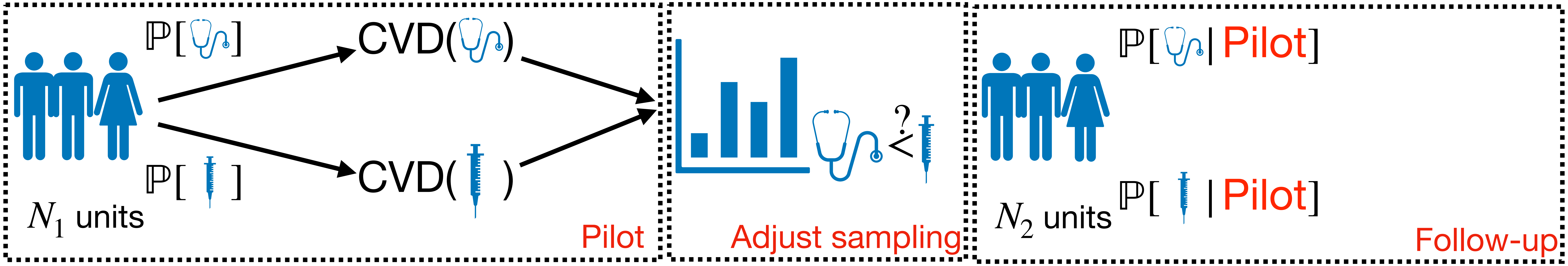

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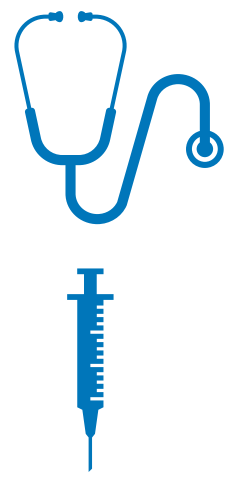

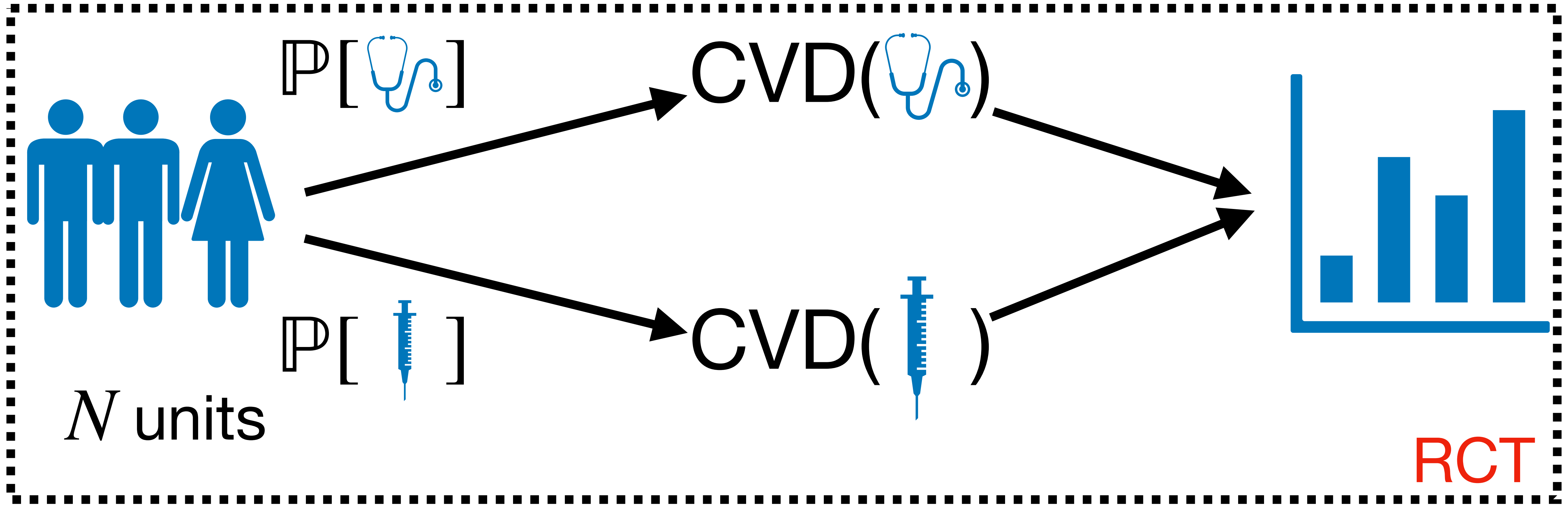

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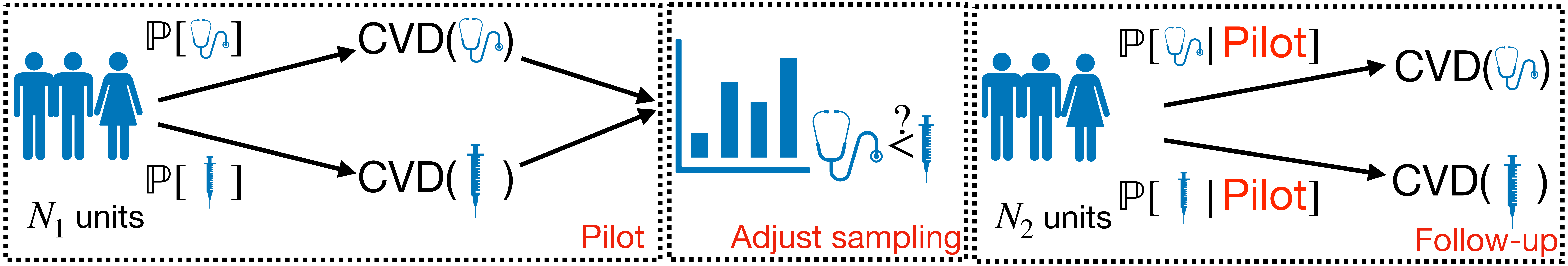

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Motivating example (cont’d)

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Adaptive sampling: adjust sampling by DIM $\equiv \widehat{\mathbb{E}}[\text{CVD}($  $)] - \widehat{\mathbb{E}}[\text{CVD}($  $)]$.

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Adaptive algorithms: $\mathbb{P}[$  $|\,\text{Pilot}] = f(\text{DIM})$.

Motivating example (cont'd)

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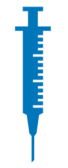
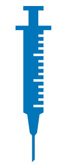
Adaptive sampling: adjust sampling by DIM $\equiv \widehat{\mathbb{E}}$ [CVD()] $-$ $\widehat{\mathbb{E}}$ [CVD()] .

Adaptive algorithms: $\mathbb{P}[$  $|$ Pilot$] = f(\mathrm{DIM})$.

- Greedy algorithm:

Motivating example (cont'd)

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Adaptive algorithms: $\mathbb{P}[$ | Pilot$] = f(\mathrm{DIM})$.

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$$\mathbb{P}[\quad | \text{Pilot}] = \begin{cases} 0.9 \text{ if DIM} > 0 \\ 0.1 \text{ if DIM} \leq 0 \end{cases}.$$

Motivating example (cont'd)

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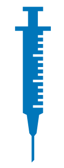
Adaptive sampling: adjust sampling by DIM $\equiv \widehat{\mathbb{E}}$[CVD()] $-$ $\widehat{\mathbb{E}}$[CVD()] .

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- (Modified) Thompson sampling:

Control : 🩺

Treatment: 💉

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Adaptive sampling: adjust sampling by $\text{DIM} \equiv \widehat{\mathbb{E}}[\text{CVD}(\text{💉})] - \widehat{\mathbb{E}}[\text{CVD}(\text{🩺})]$.

Adaptive algorithms: $\mathbb{P}[\text{💉} \mid \text{Pilot}] = f(\text{DIM})$.

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- (Modified) Thompson sampling:

$\mathbb{P}$[ | Pilot] = Φ(DIM) .

- Interim p-value: define $T = \text{DIM}/\hat{\sigma}$ with $p_L = \Phi(T)$ and $p_R = 1 - p_L$.

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$$\mathbb{P}[\;\mid \text{Pilot}] = 0.9 \text{ if } p_L \leq \alpha;$$

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$\mathbb{P}[$  $\mid$ Pilot$] = 0.9$ if $p_L \leq \alpha; \quad 0.1$ if $p_L \geq 1 - \alpha;$

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$\mathbb{P}$[ | Pilot] $= 0.9$ if $p_L \leq \alpha$; $\quad 0.1$ if $p_L \geq 1 - \alpha$; $\quad 0.5$ if $p_L \in (\alpha, 1 - \alpha)$.

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Post-experiment evaluation

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Control : 

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Decision-related questions:

Post-experiment evaluation

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- Is the new treatment more effective? ($\mathbb{E}[\text{Diff}] \equiv \mathbb{E}[\text{CVD}($  $) - \text{CVD}($$)] < 0$?)

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Post-experiment evaluation

Decision-related questions:

- Is the new treatment more effective? ($\mathbb{E}[\text{Diff}] \equiv \mathbb{E}[\text{CVD}($  $) - \text{CVD}($$)] < 0$?)
- Is the treatment effect significant? ($\mathbb{E}[\text{CVD}($  $)] < 0.03$?)

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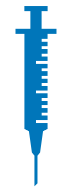
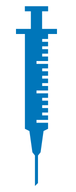
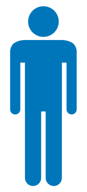
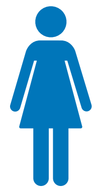
- Is the new treatment more effective? ($\mathbb{E}[\text{Diff}] \equiv \mathbb{E}[\text{CVD}($  $) - \text{CVD}($  $)] < 0$?)
- Is the treatment effect significant? ($\mathbb{E}[\text{CVD}($  $)] < 0.03$?)
- Is the treatment more effective for particular subgroup? ($\mathbb{E}[\text{Diff} \mid$  $] > \mathbb{E}[\text{Diff} \mid$  $]$?)

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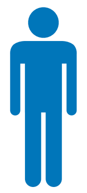
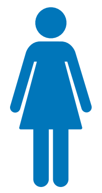
- Is the treatment more effective for particular subgroup? (𝔼[Diff | ] > 𝔼[Diff | ]?)

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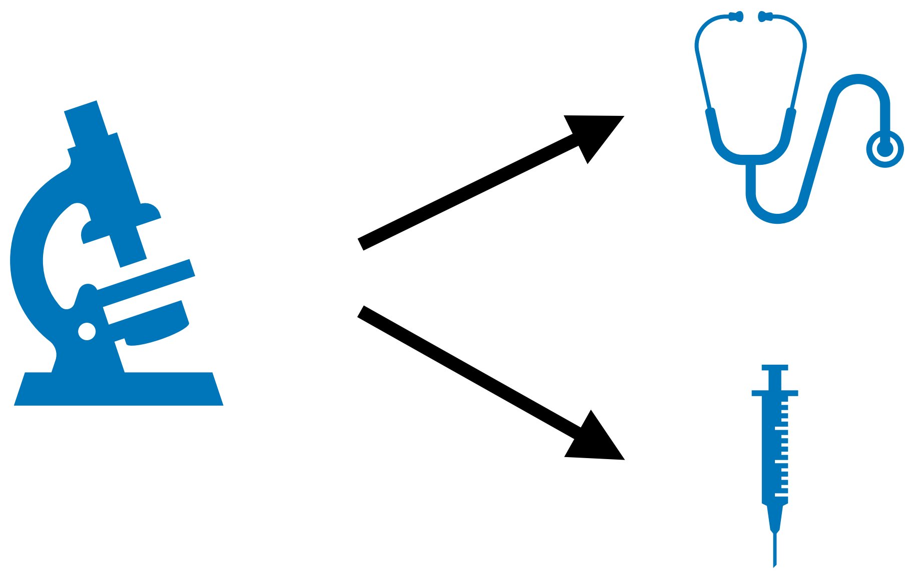

Data collection: treatment A, outcome Y.

Stage 1: N_1 sample, $(A_1^{(1)}, Y_1^{(1)}), \ldots, (A_{N_1}^{(1)}, Y_{N_1}^{(1)})$.

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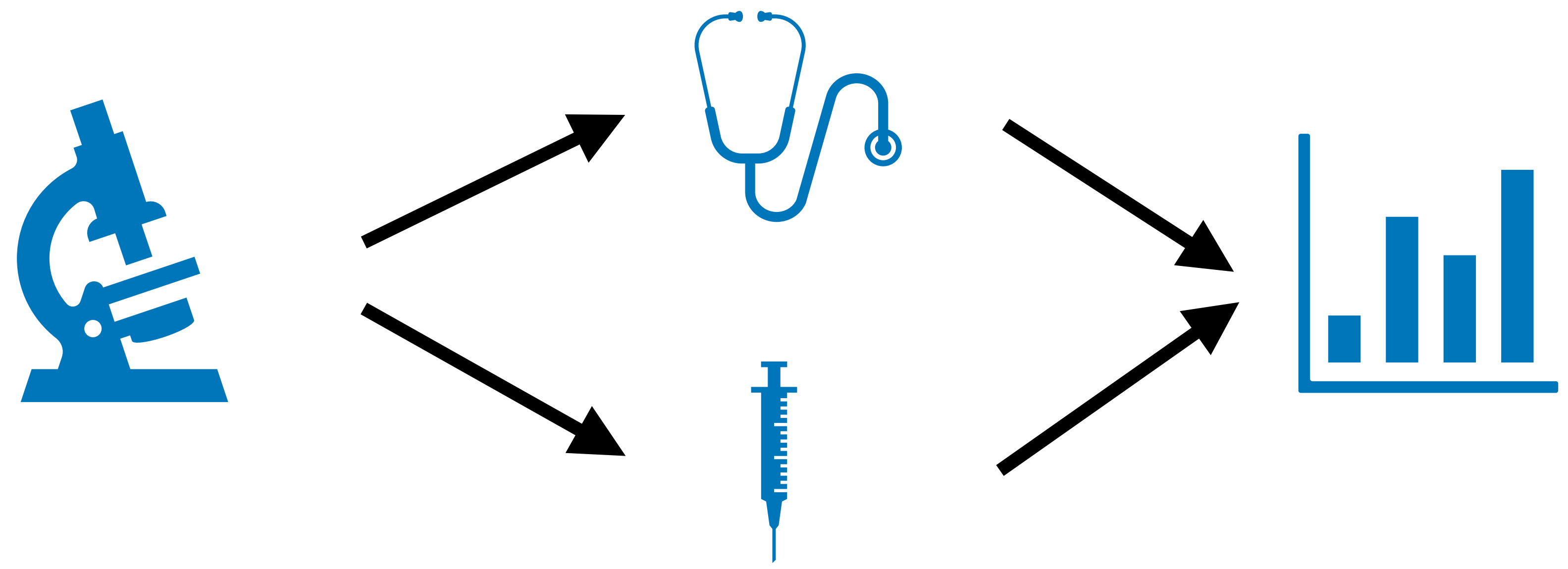

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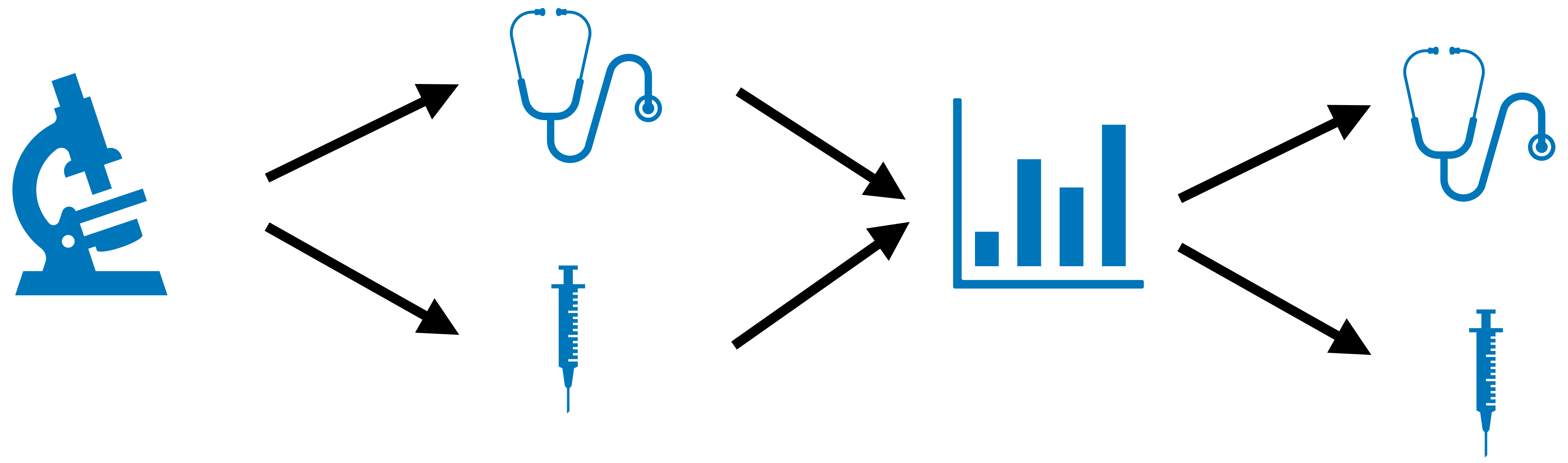

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Stage 2: N_2 sample, $(A_1^{(2)}, Y_1^{(2)}), \ldots, (A_{N_2}^{(2)}, Y_{N_2}^{(2)})$.

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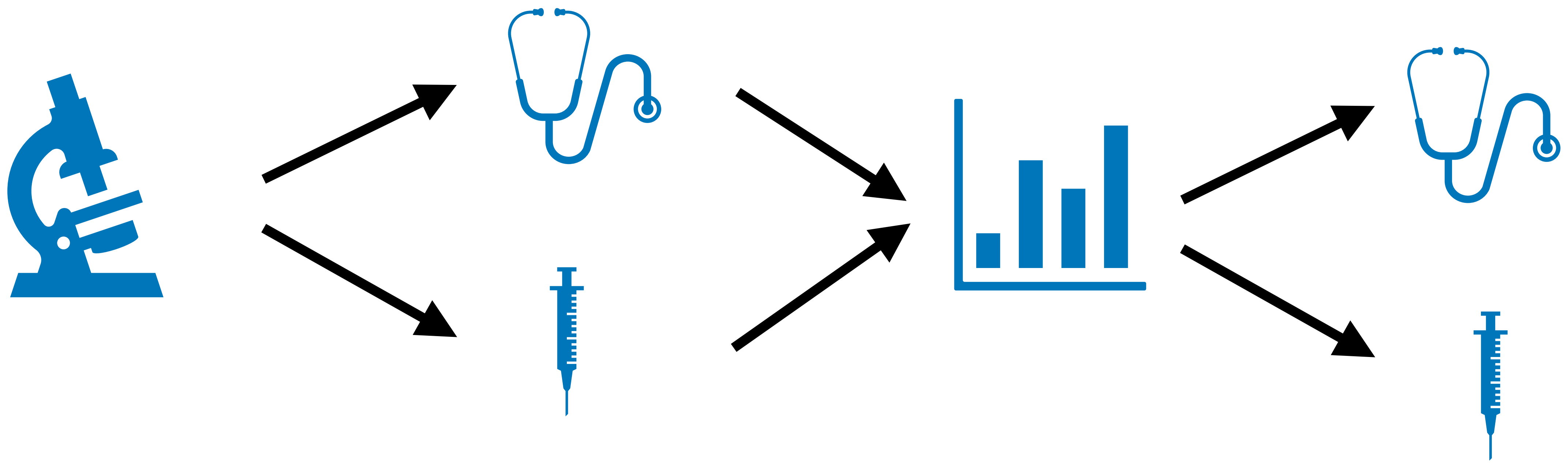

Data collection: treatment A, outcome Y.

Stage 1: N_1 sample, $(A_1^{(1)}, Y_1^{(1)}), \ldots, (A_{N_1}^{(1)}, Y_{N_1}^{(1)})$.

Adaptive sampling: greedy algorithm, Thompson sampling, interim p-value…

Stage 2: N_2 sample, $(A_1^{(2)}, Y_1^{(2)}), \ldots, (A_{N_2}^{(2)}, Y_{N_2}^{(2)})$.

Challenge: pooled data $(A_u^{(t)}, Y_u^{(t)})_{u \in [N_t], t \in [2]}$ are highly dependent!

Literature review

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Conditional inference: conditional on output of adaptive algorithm.

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- Data-splitting/data-carving: Chen and Andrews (2023).

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- Data-splitting/data-carving: Chen and Andrews (2023).
- Randomization inference: Freidling et al. (2024).

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Conditional inference: conditional on output of adaptive algorithm.

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Can we explicitly characterize these non-normal weak limits?

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2. **Methodologically:** A fast bootstrap procedure for sampling from the limiting distribution and apply it for hypothesis testing.

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$$\text{IPW}^{(t)}(s) \equiv \frac{1}{N_t} \frac{\sum_{u=1}^{N_t} \mathbf{1}(A_u^{(t)} = s) Y_u^{(t)}}{e_N^{(t)}(s)} \qquad \text{WIPW}(s) \equiv \sum_{t=1}^{2} \frac{(e_N^{(t)}(s))^m}{\sum_{t=1}^{2} (e_N^{(t)}(s))^m} \text{IPW}^{(t)}(s)$$

Special cases:

- $m = 0$: familiar IPW estimator.
- $m = 1/2$: variance-stabilizing IPW estimator.

Our focus:

Weighted IPW (WIPW) estimator, $\widehat{\mathbb{E}}(Y_u^{(t)}) = 0$

$m \in \{0, 1/2\}$

A peek at the sampling distribution

A peek at the sampling distribution

Simulation setup:

A peek at the sampling distribution

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- $Y_u(0) \sim N(0,1),\ Y_u(1) \sim N(-c_N/\sqrt{N}, 9)$ where $c_N \in \{0, -5, -10, -15\}$.

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$$N = 1000,\ N_1 = N_2 = 500$$

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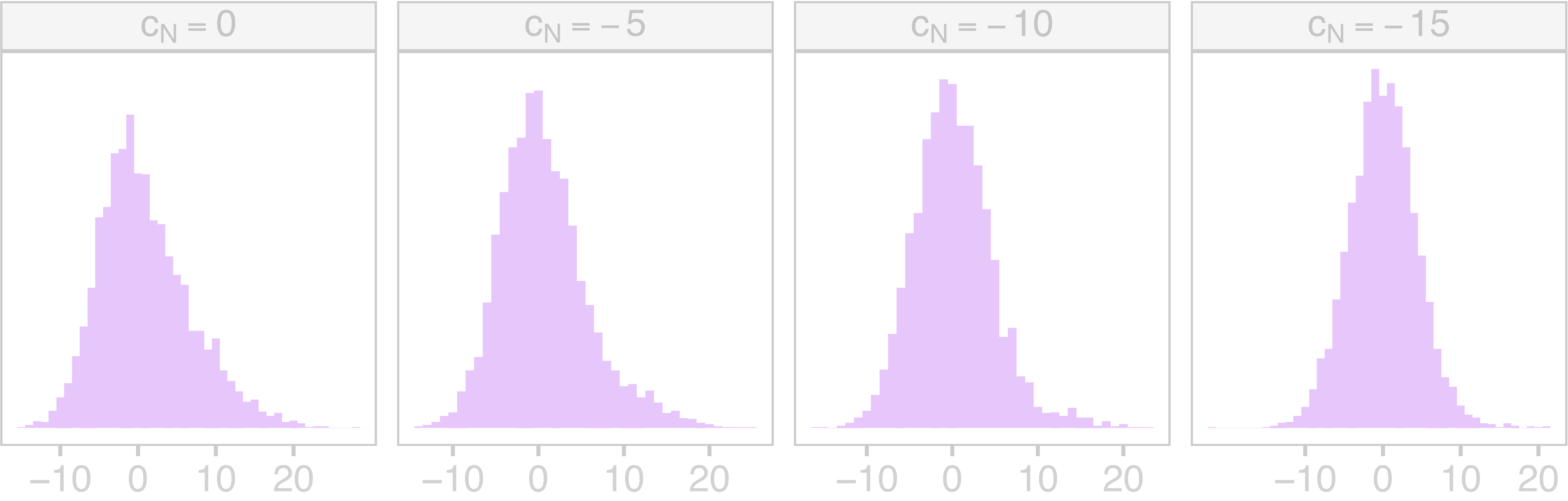

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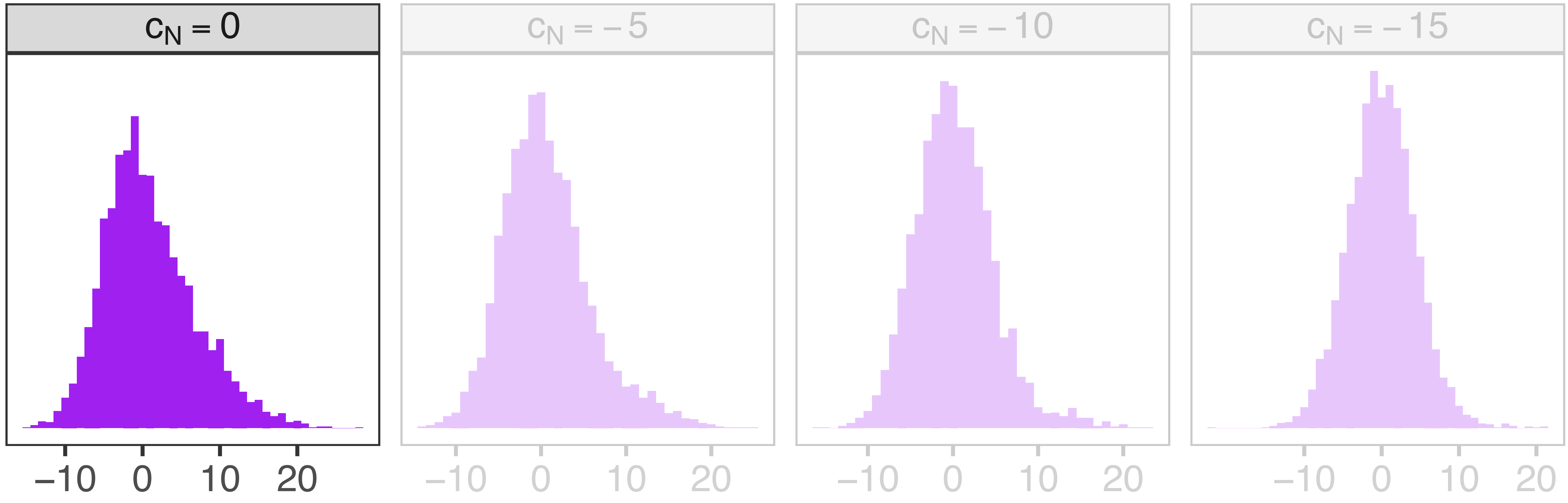

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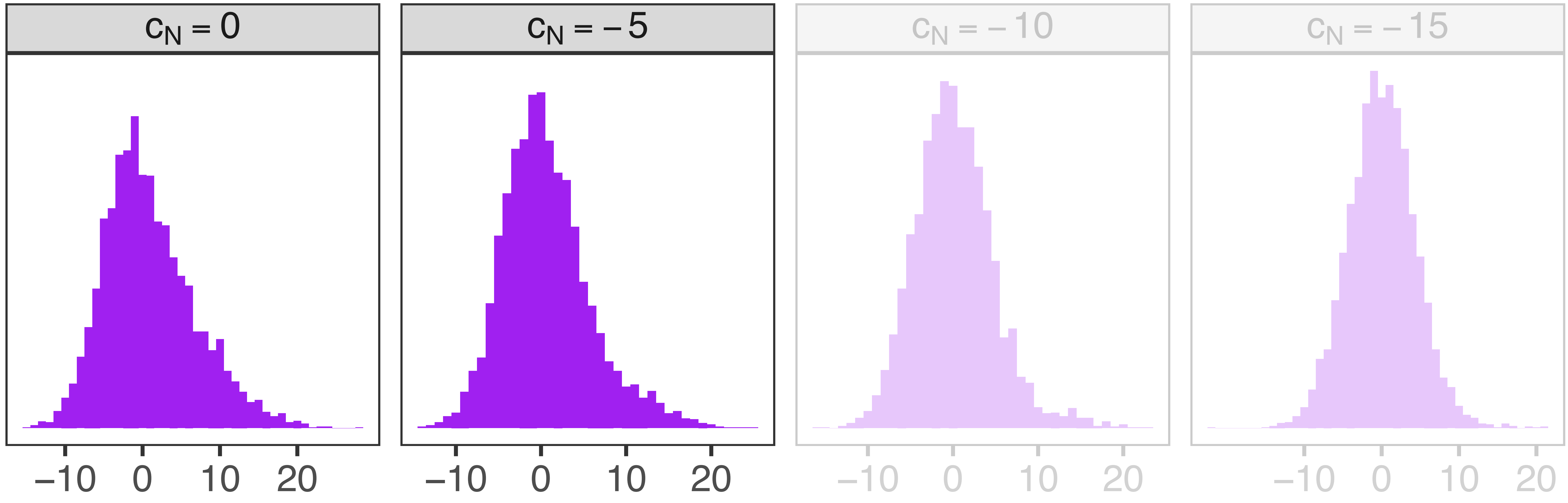

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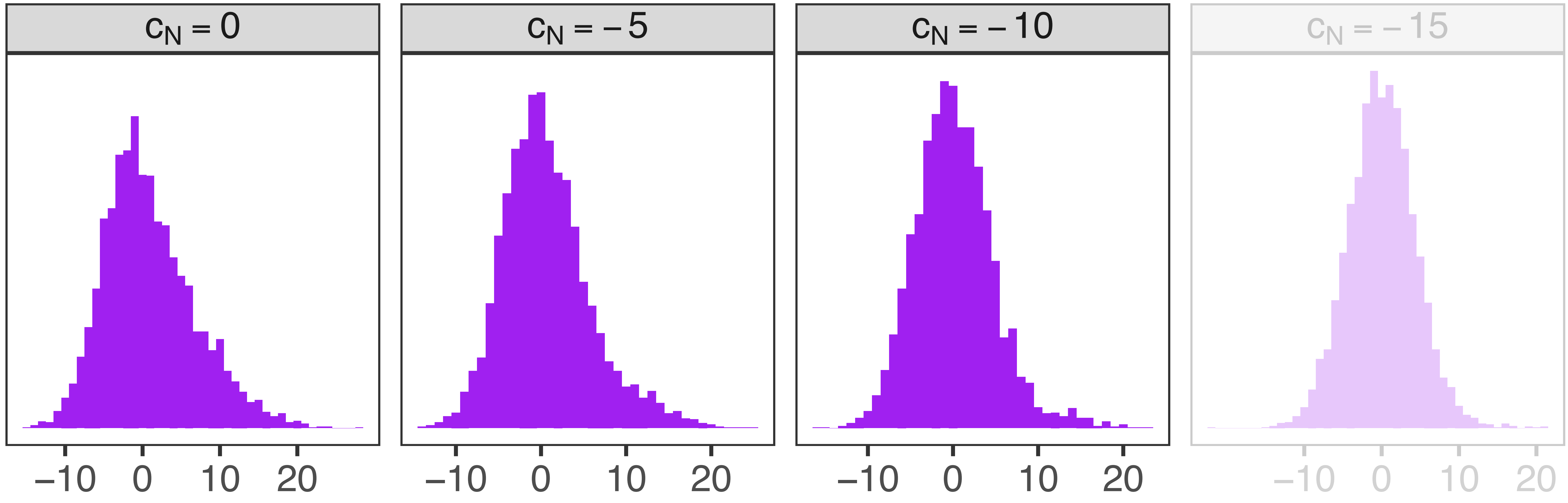

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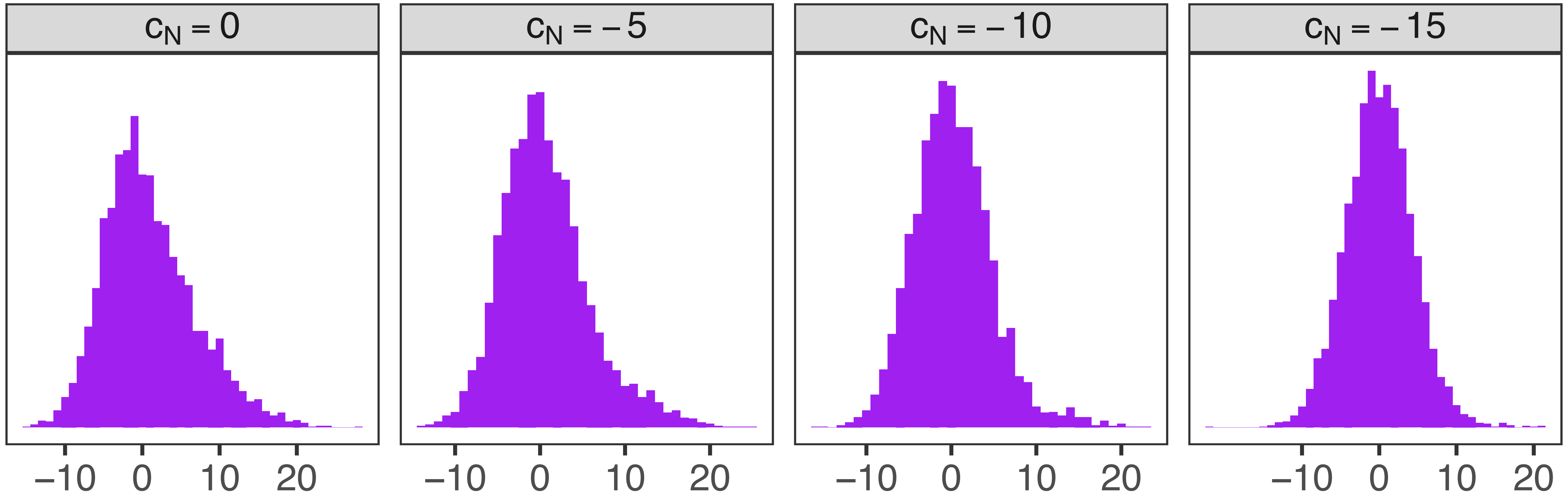

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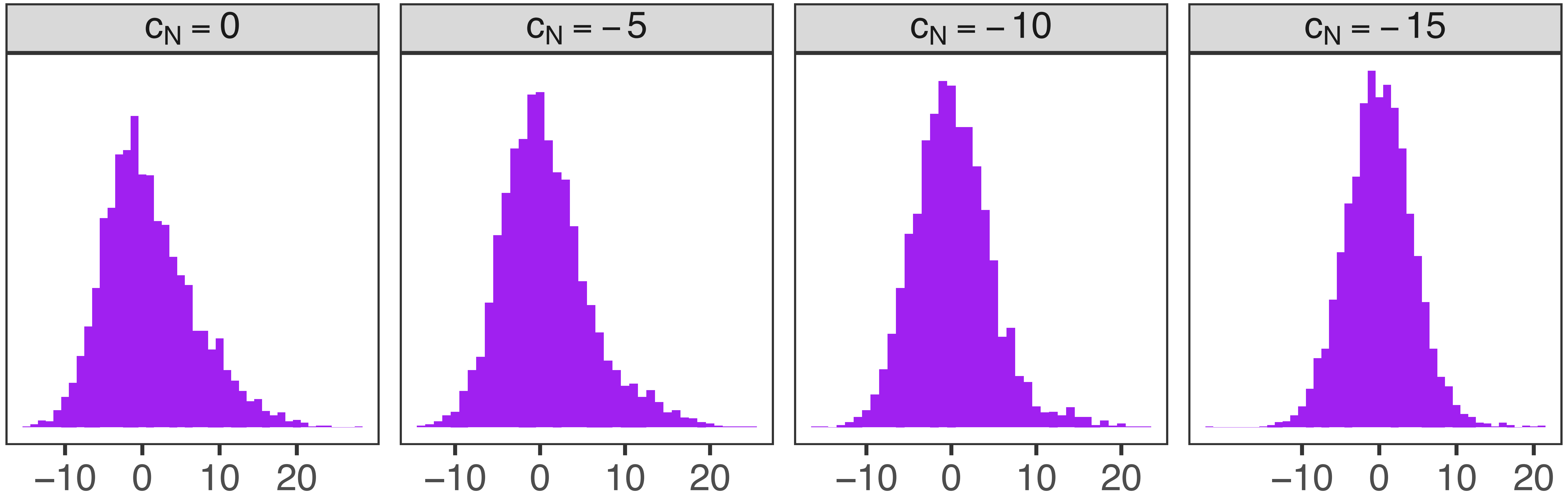

Weak signal

A peek at the sampling distribution

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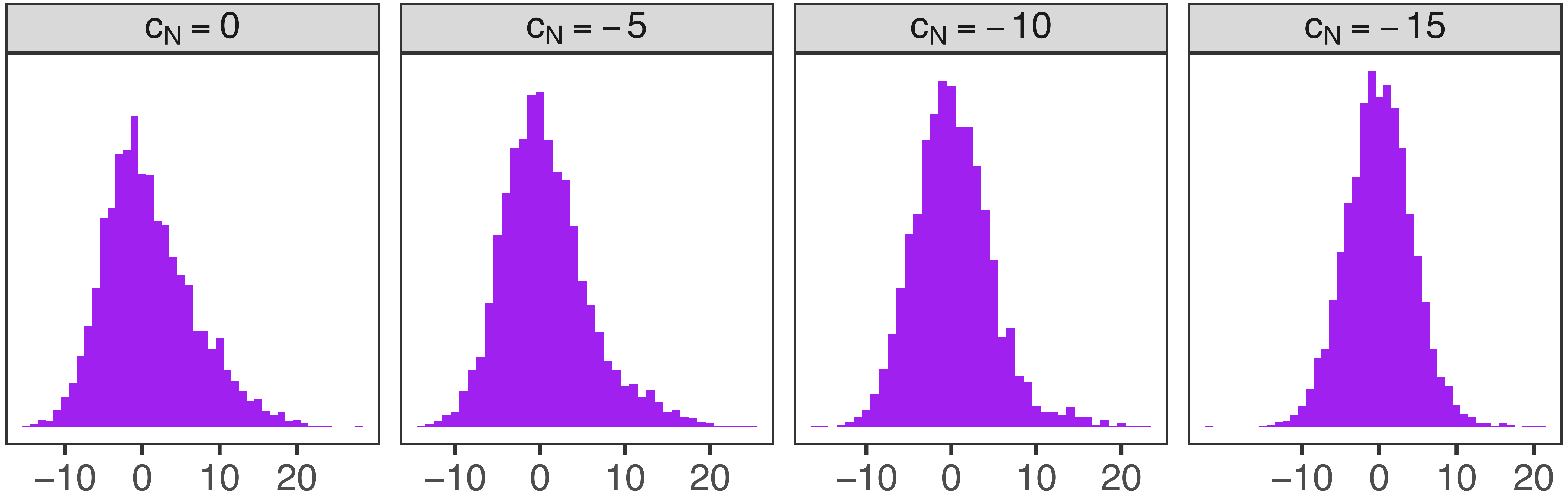

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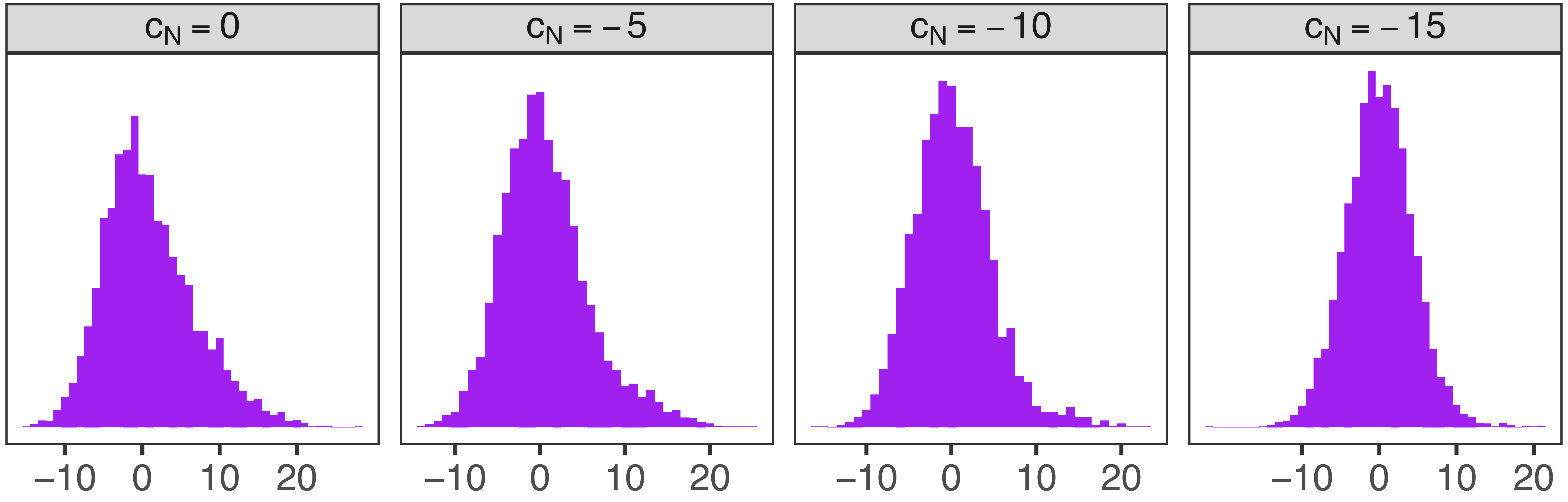

Weak signal → Strong signal

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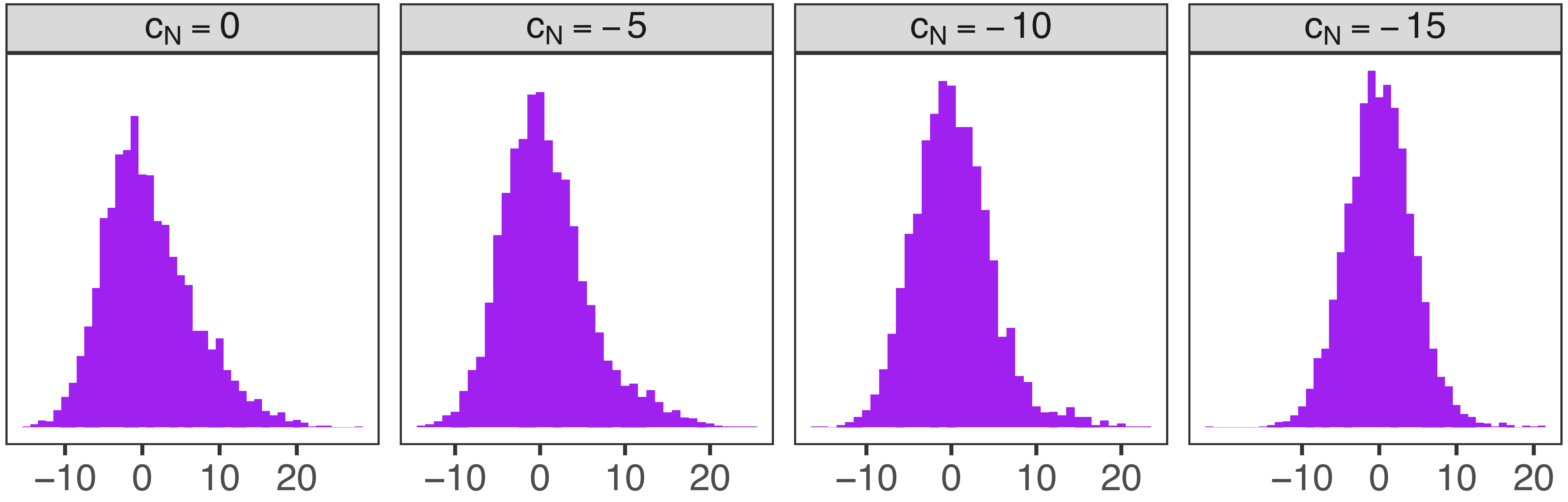

Weak signal → **Strong signal**

What role does signal strength play here?

New weak limits for WIPW

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- Highly nontrivial proof (requires extend some classical normal approximation results).[1]

1: Chatterjee and Meckes (2008)

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Comparison to Hirano and Porter (2023):

- No requirement on potential outcome distribution.
- Transparency in assumption.
- Tailored towards to a class of estimators.

Comparison to Hadad et al. (2021):

- Same test statistic considered in both papers ($m = 1/2$).
- Full-spectrum signal strength.
- Tailored towards to two-stage experiments.

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Asymptotic

Sampling

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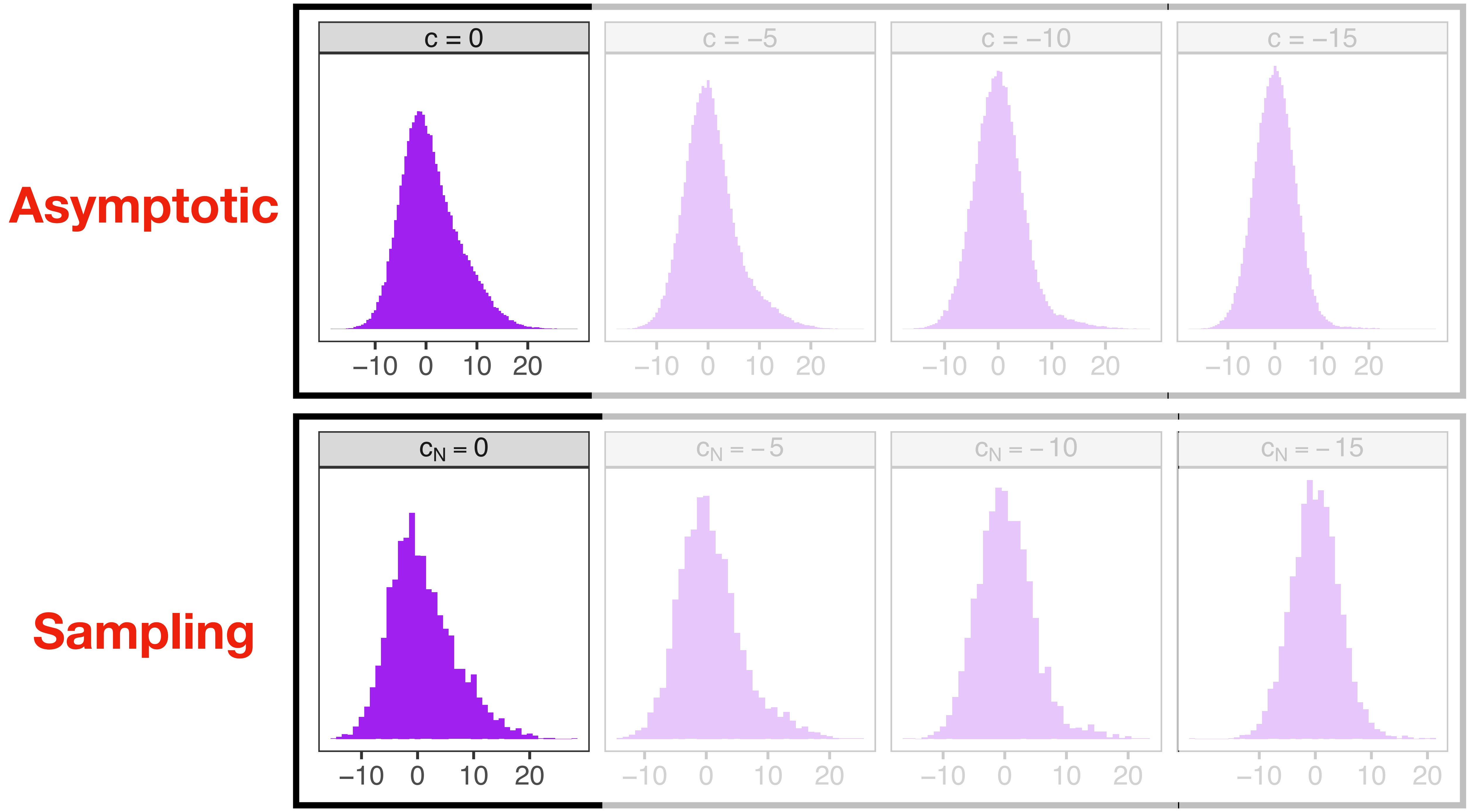

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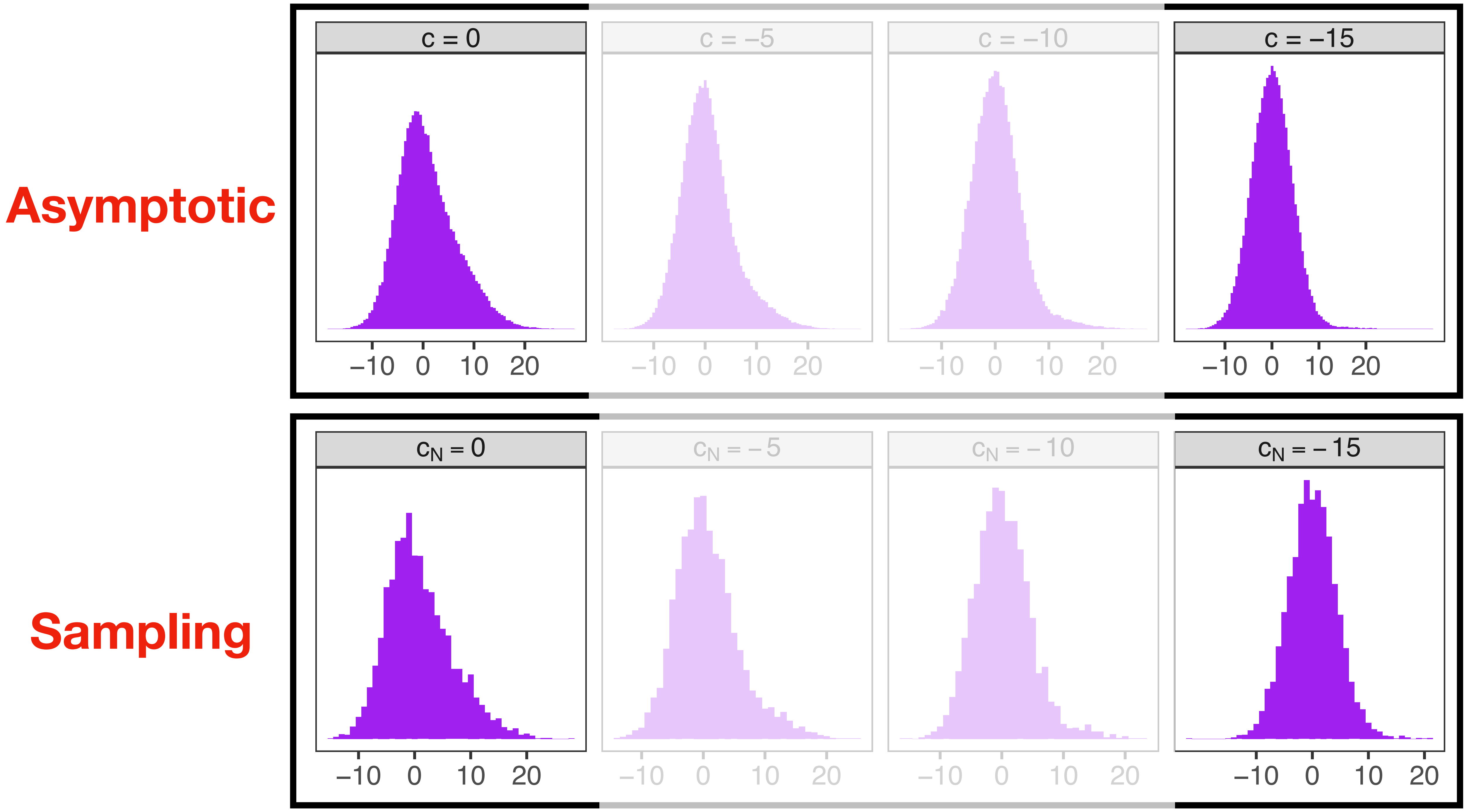

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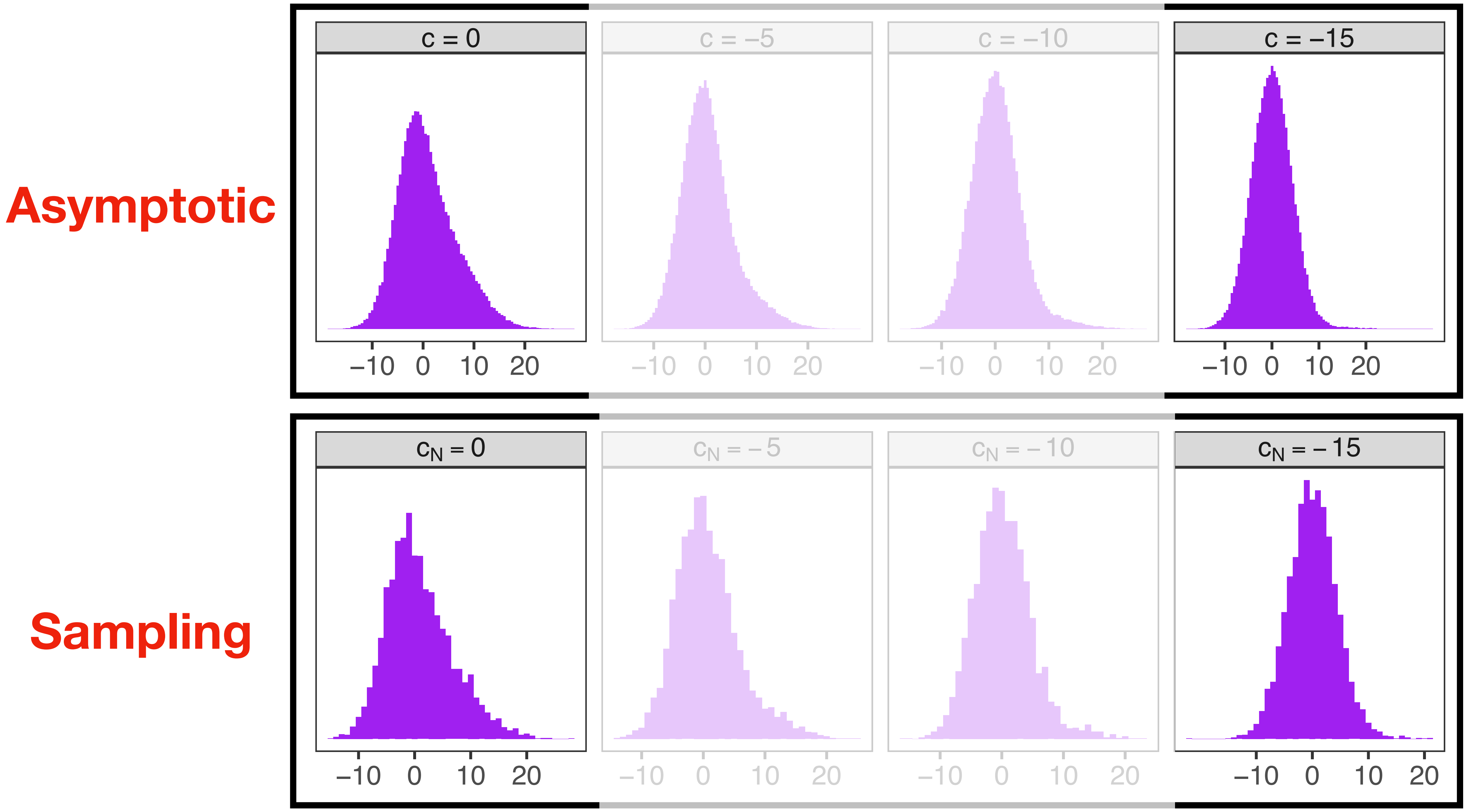

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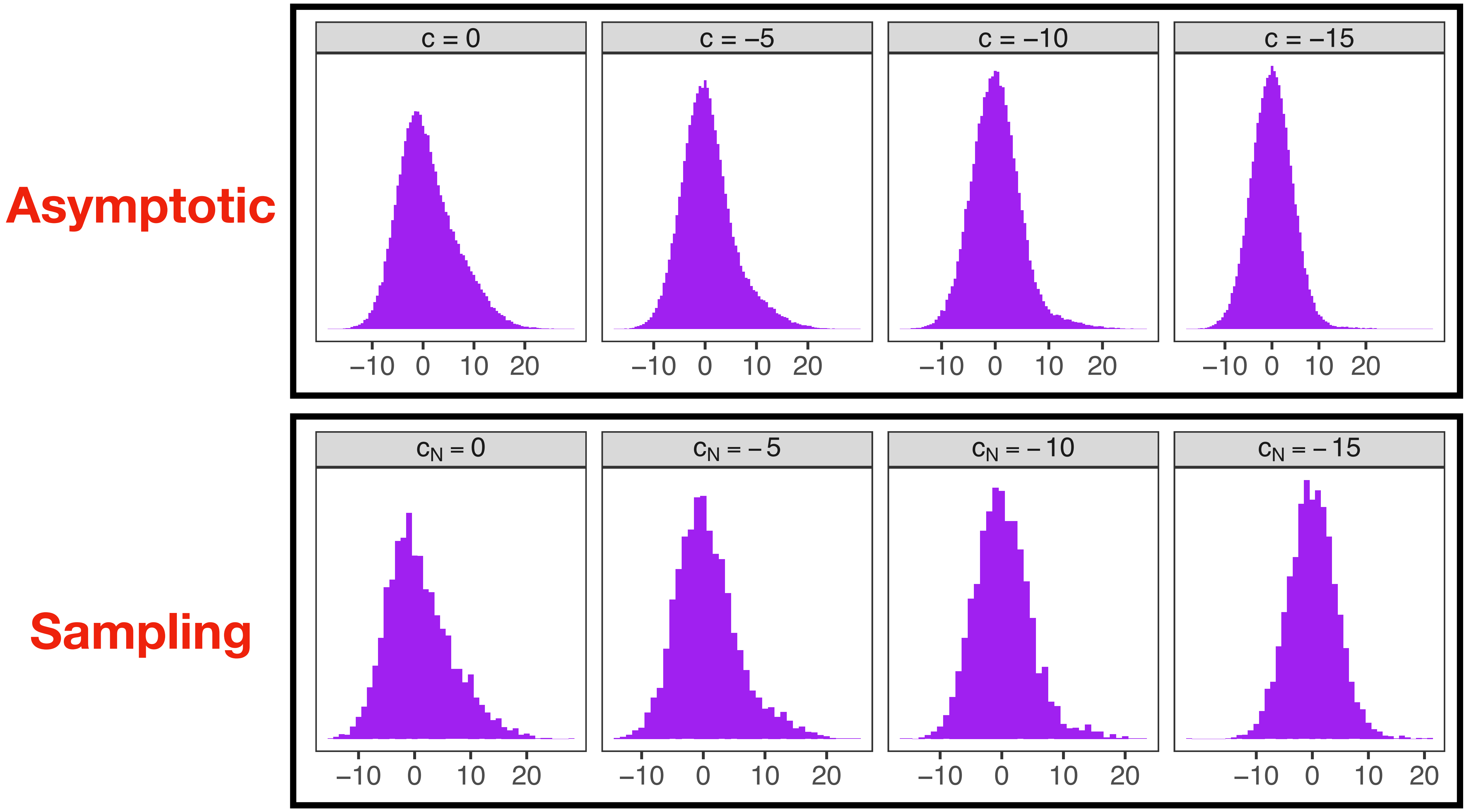

Implication on hypothesis testing

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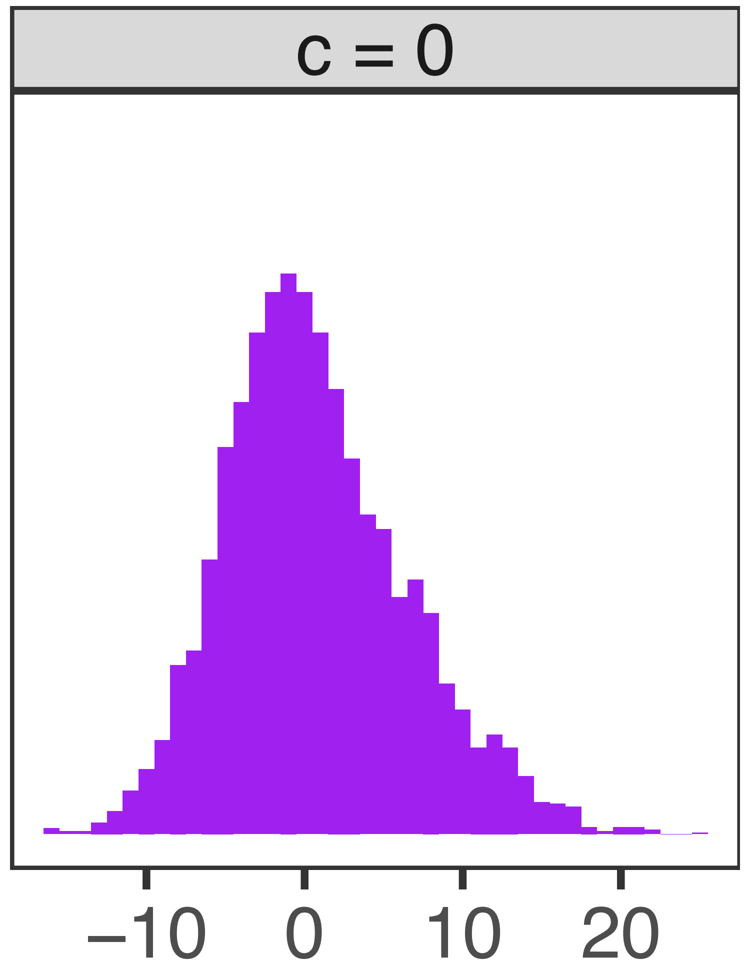

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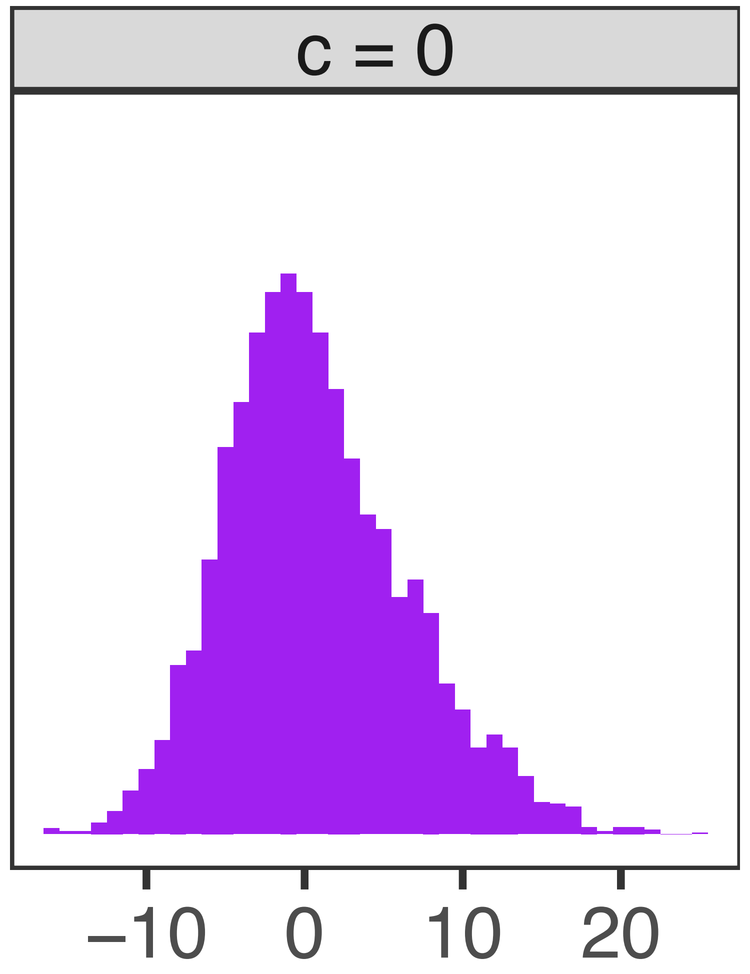

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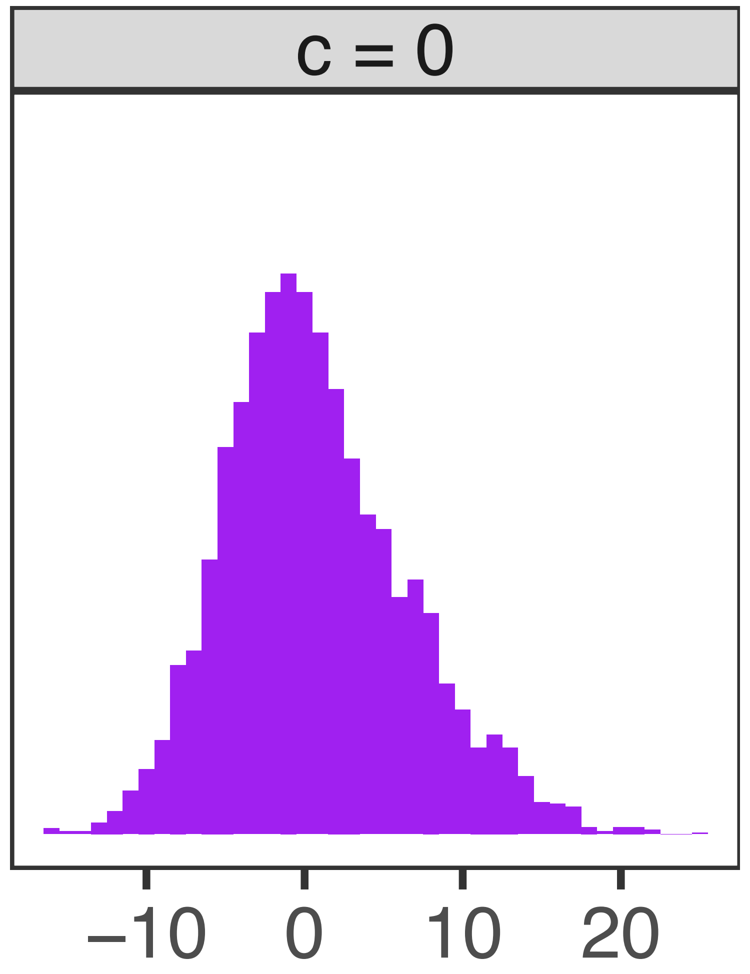

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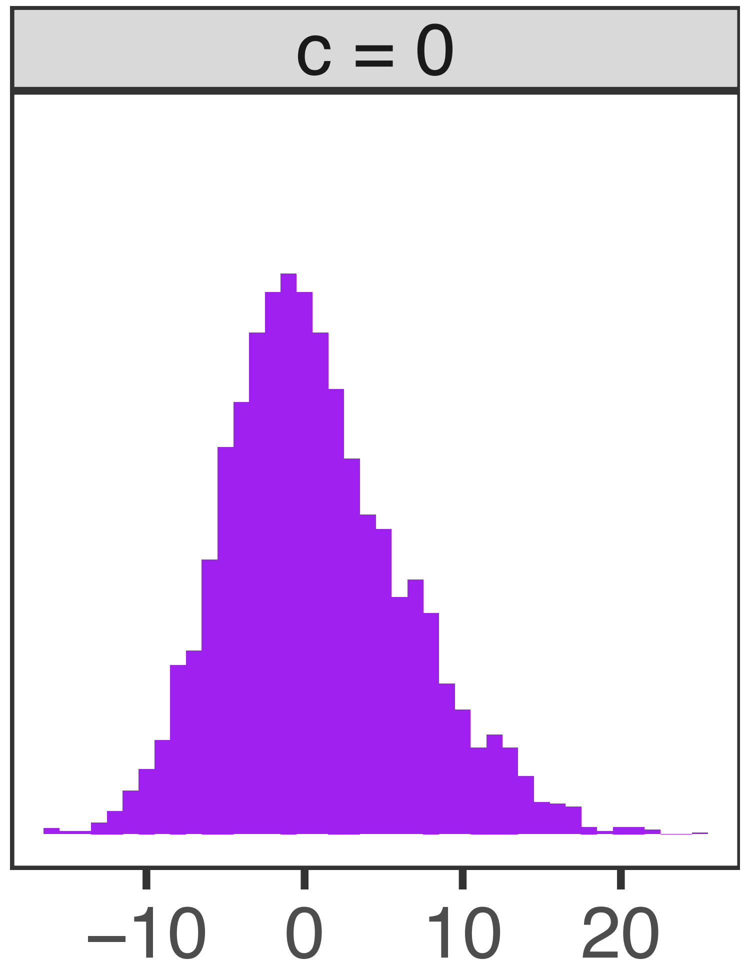

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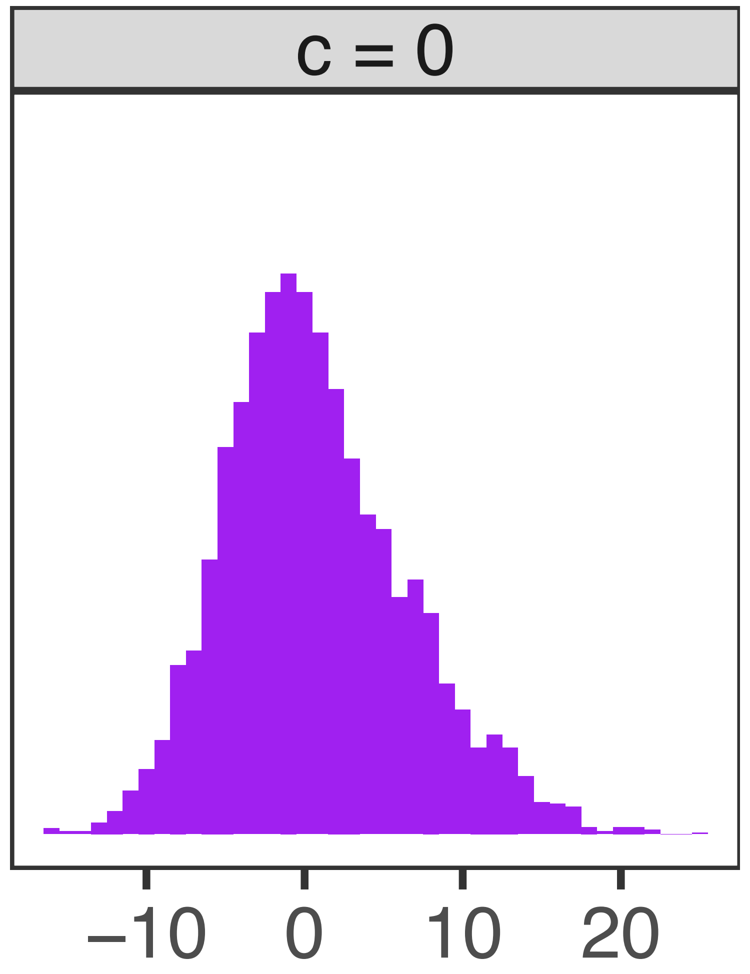

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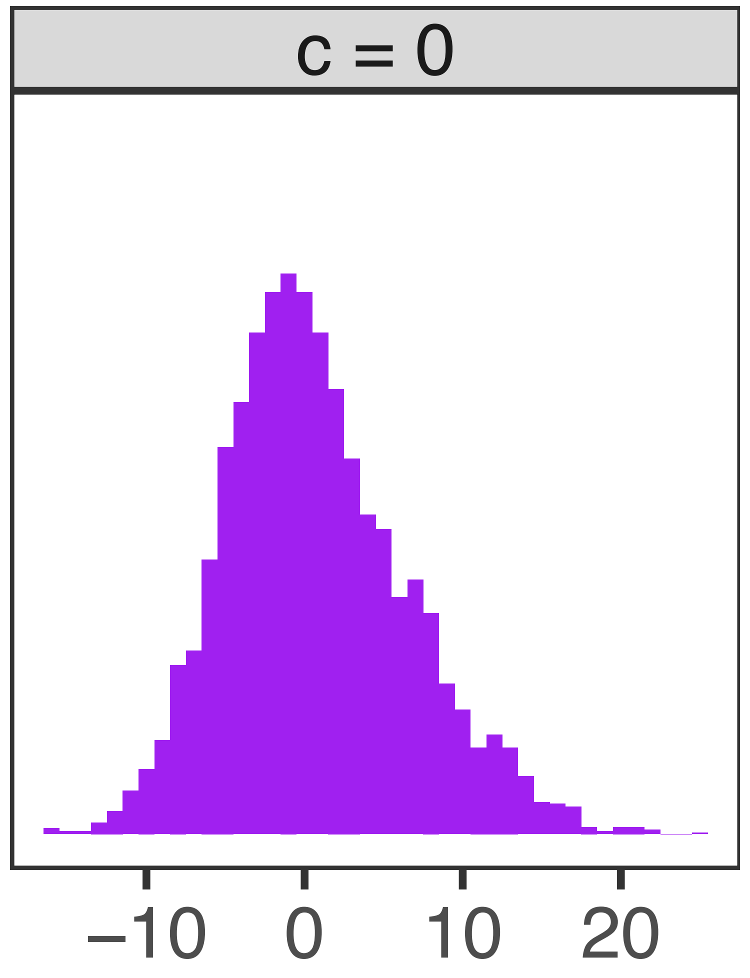

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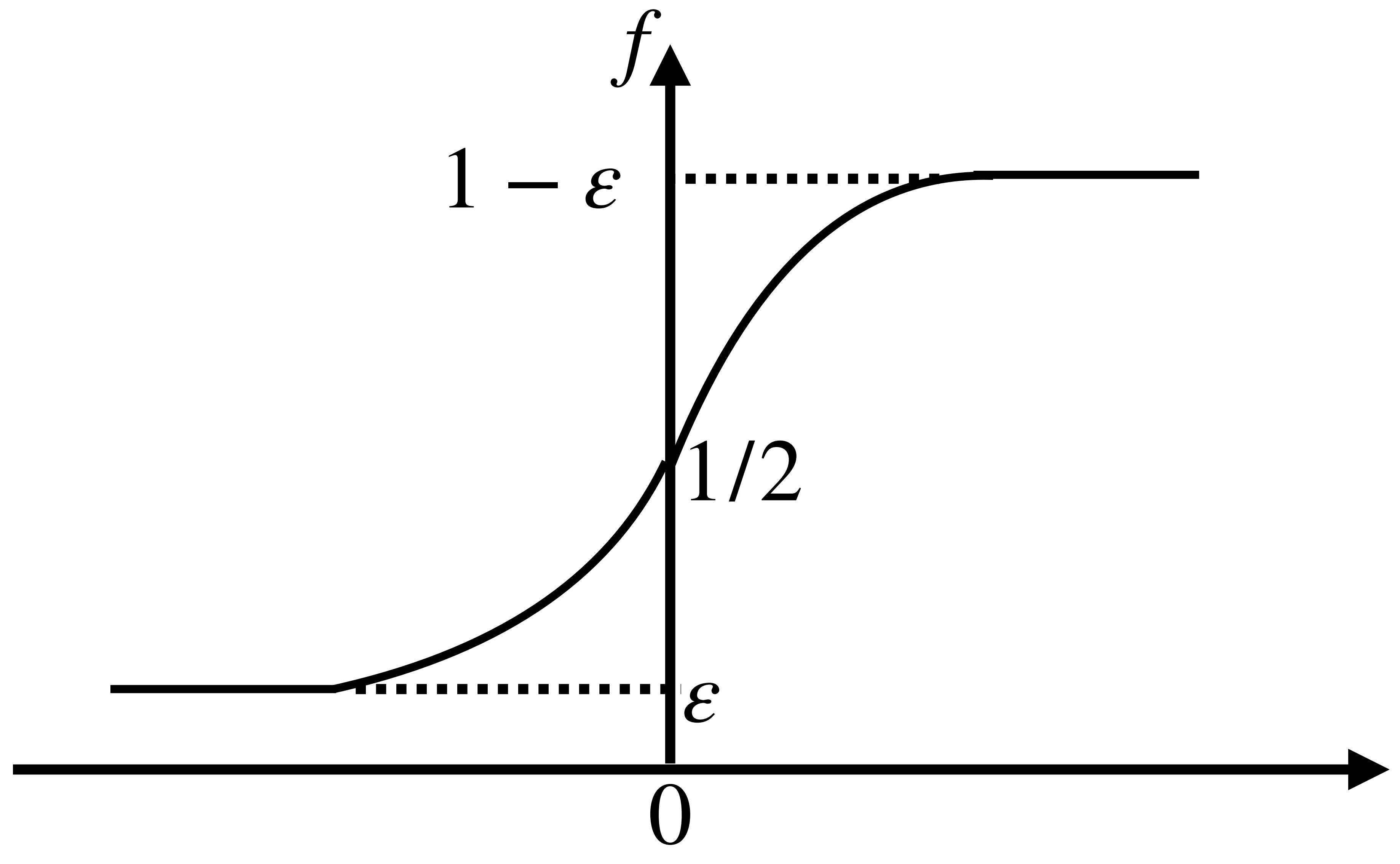

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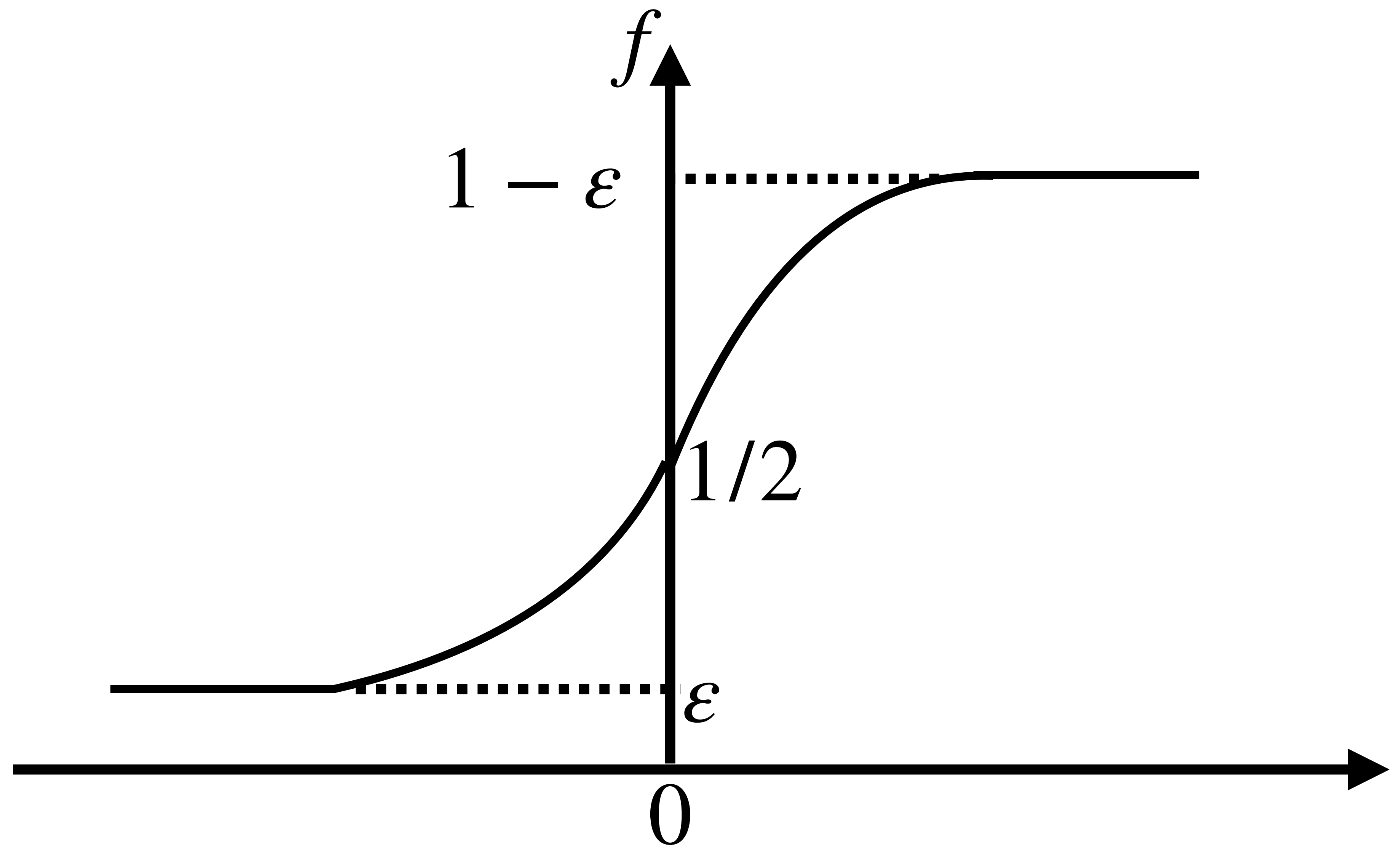

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Numerical simulation

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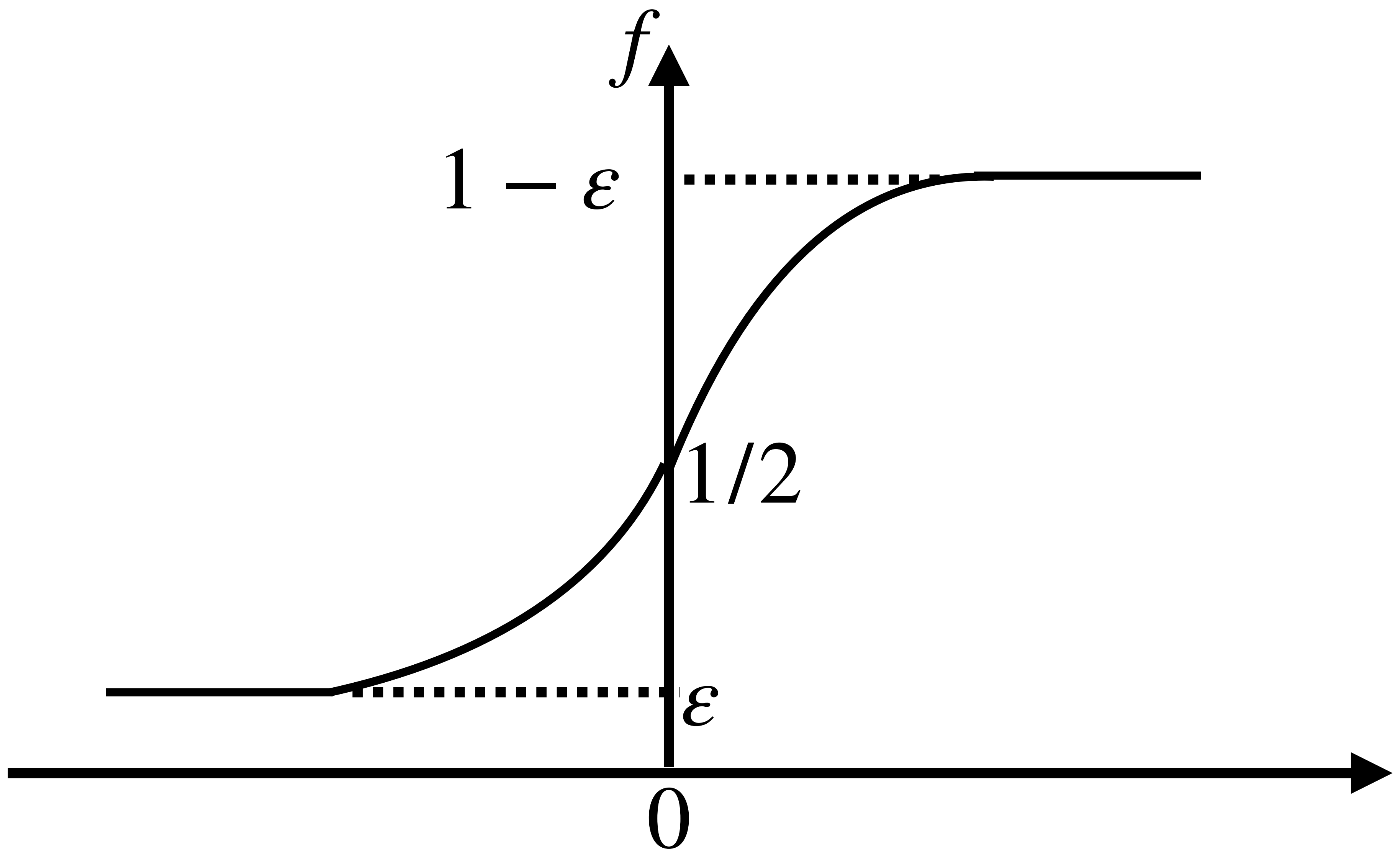

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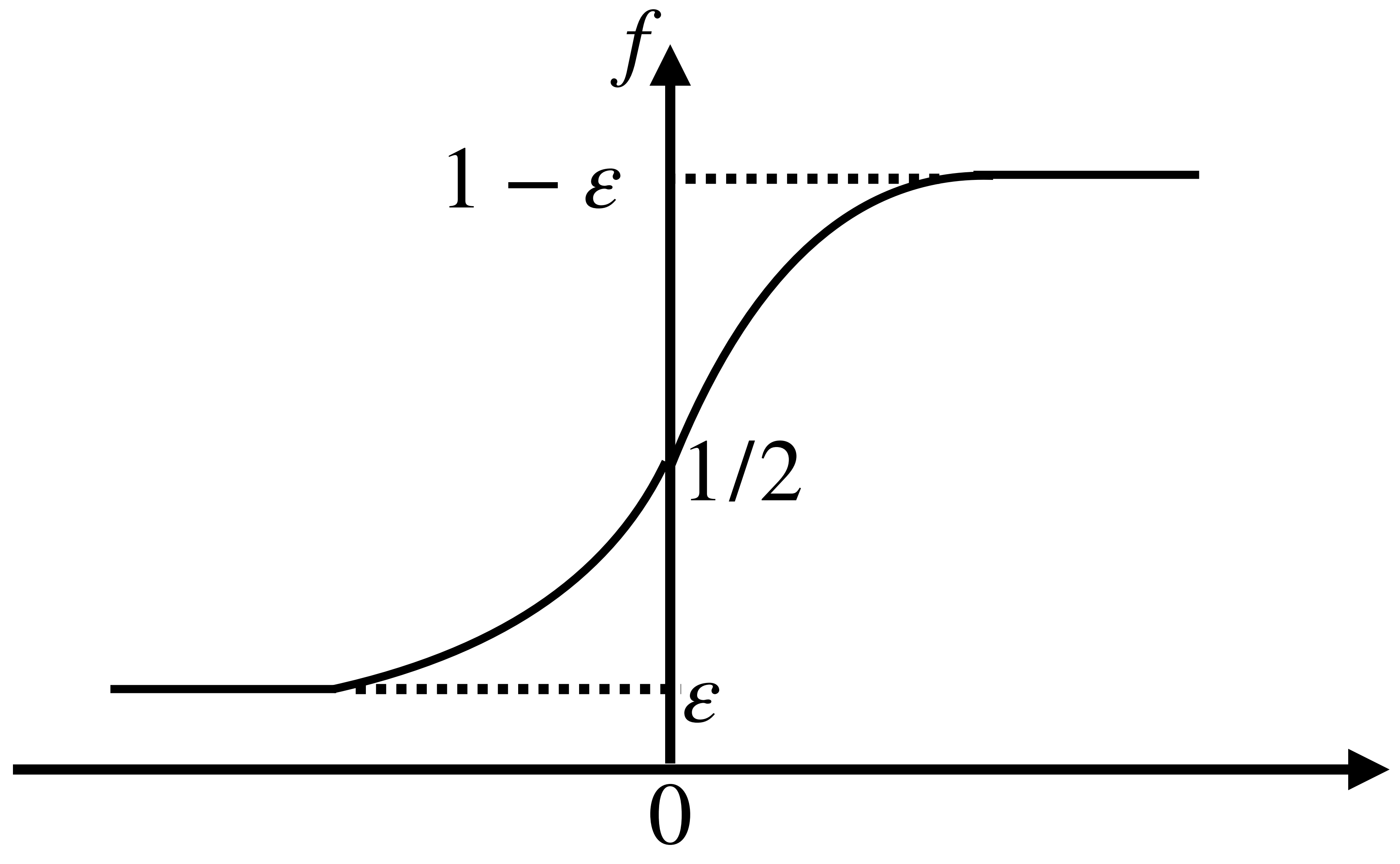

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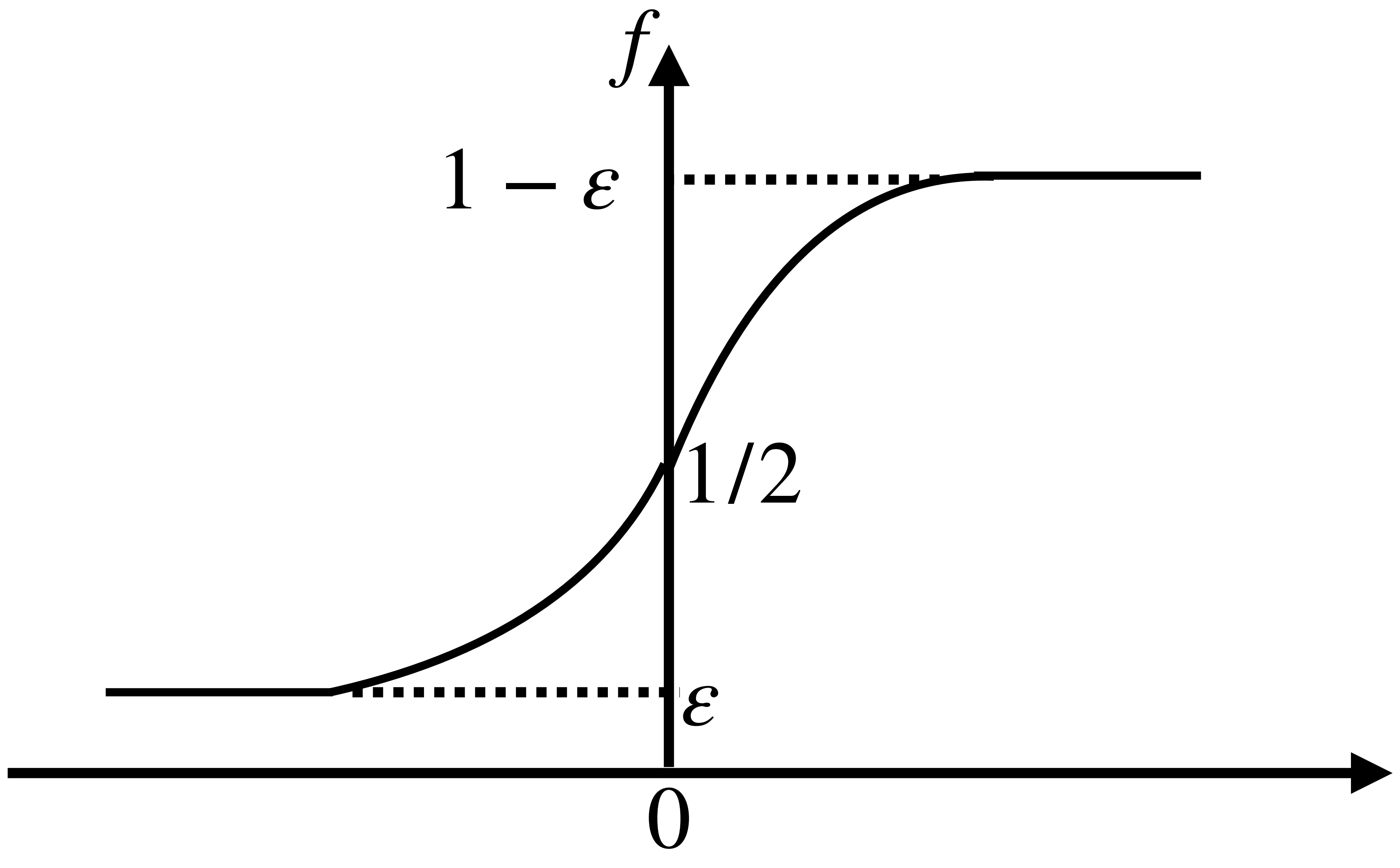

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3. Sample-splitting with IPW estimator.

Numerical simulation

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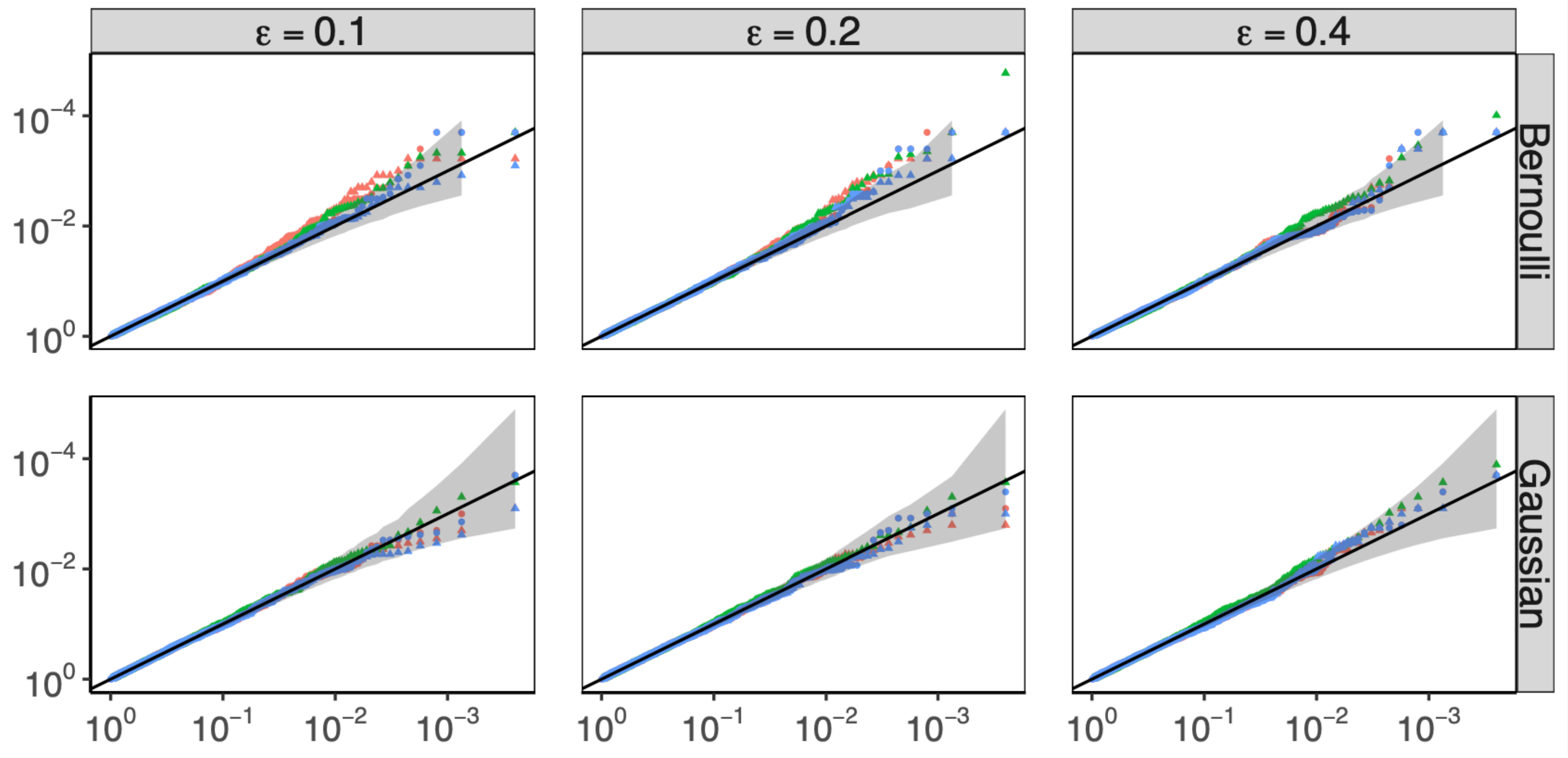
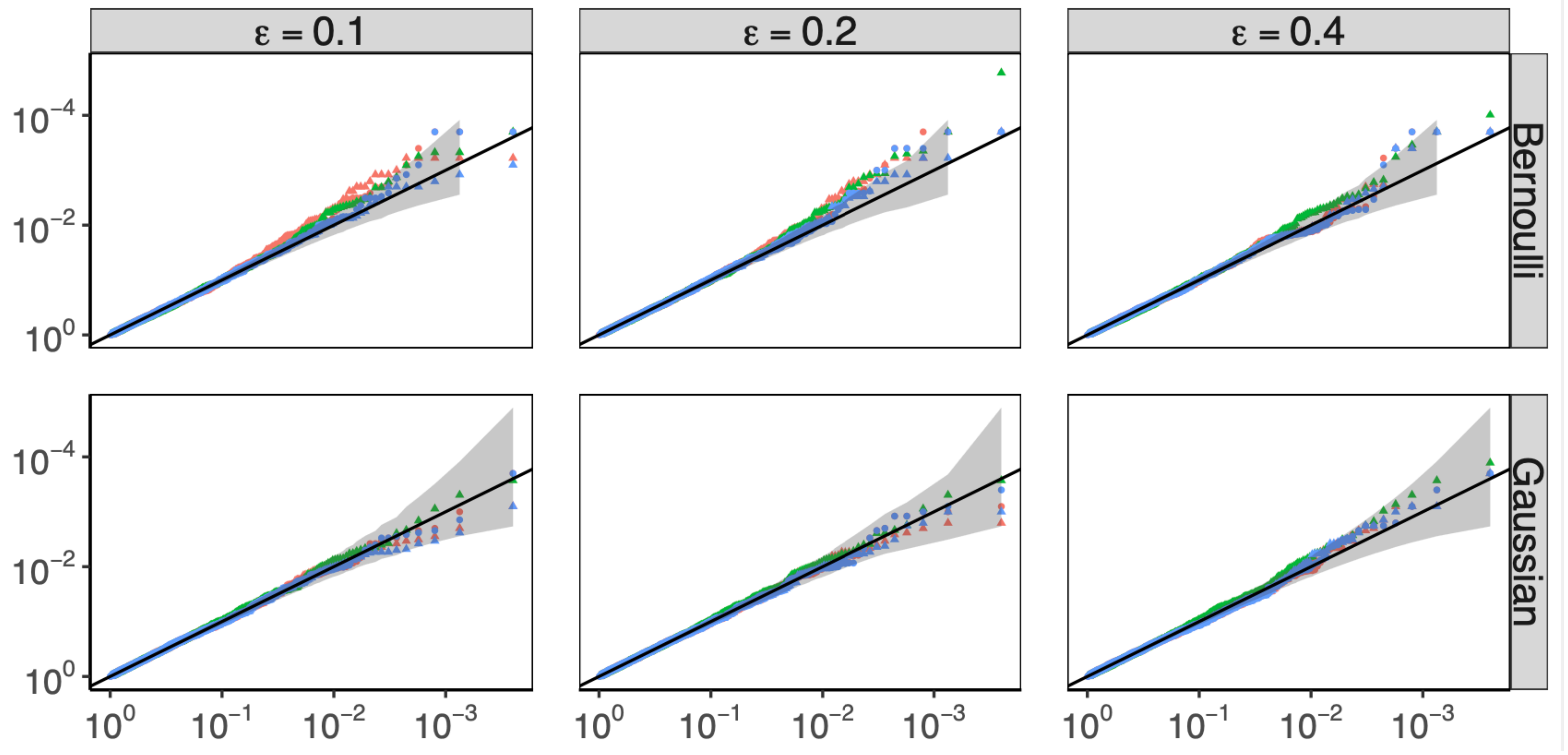


$\theta = 0$: null hypothesis

Numerical simulation

Main messages:



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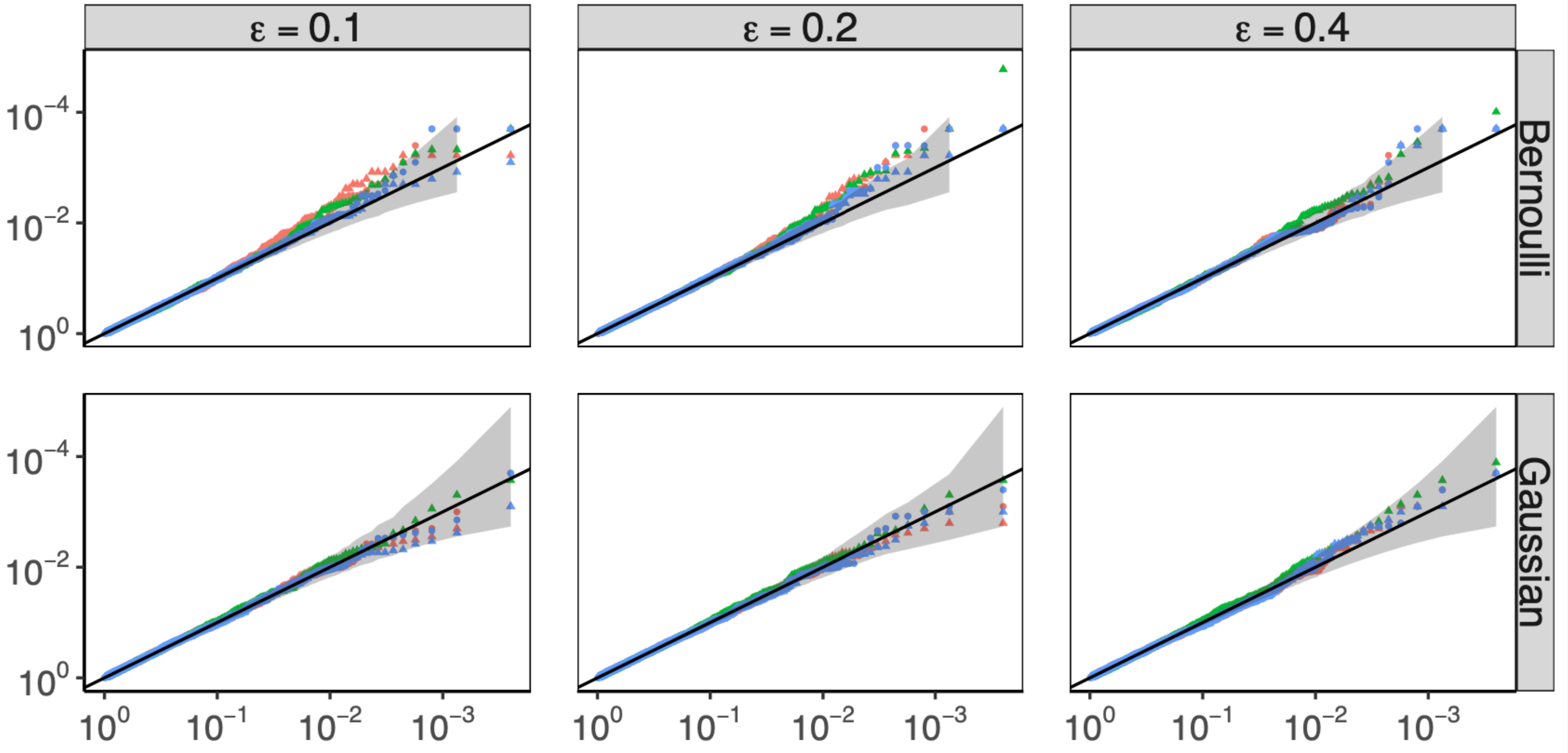

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Normalized
Unnormalized
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$m = 0$ (IPW)
$m = 1/2$ (variance-stablizing IPW)

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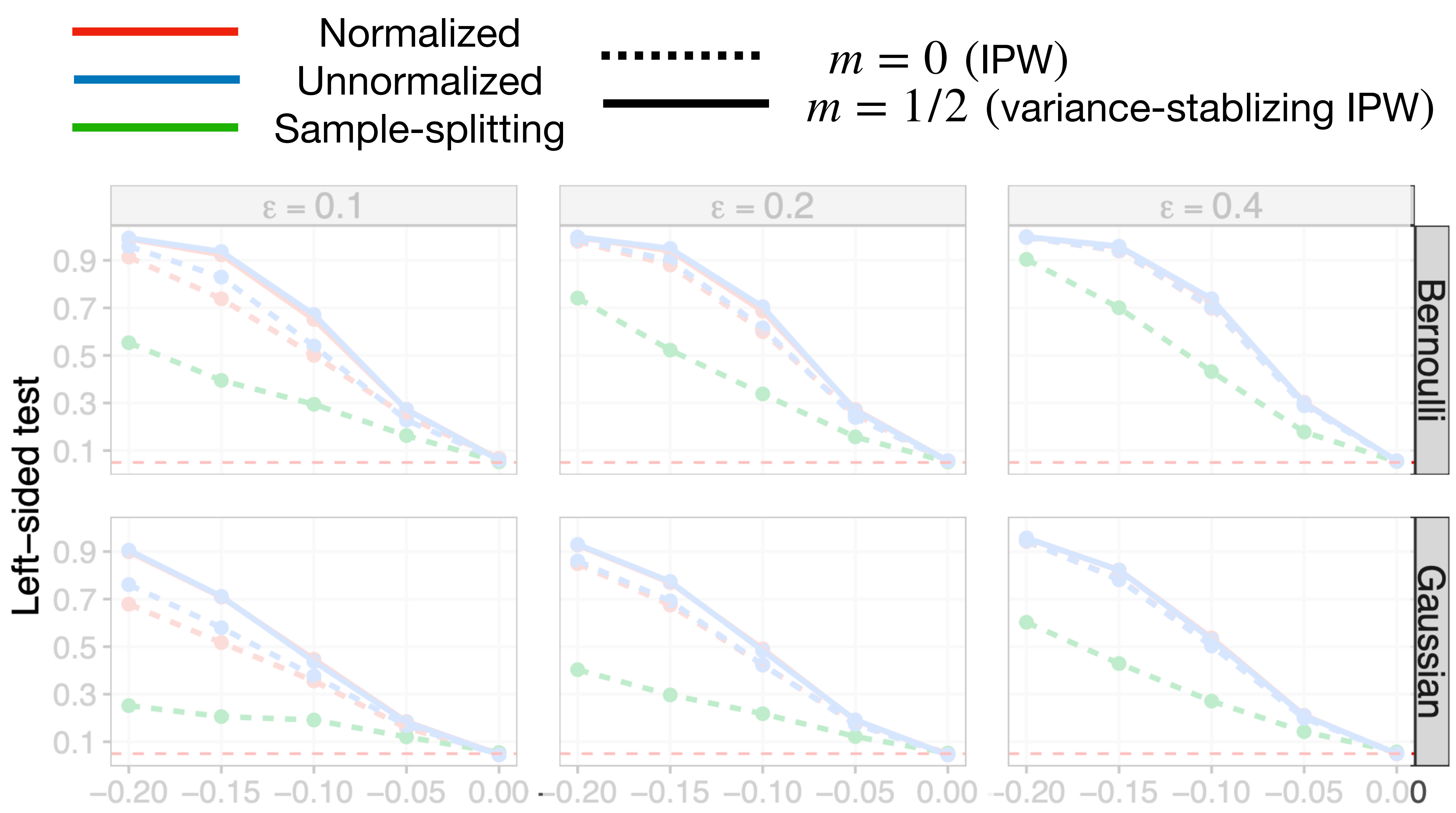

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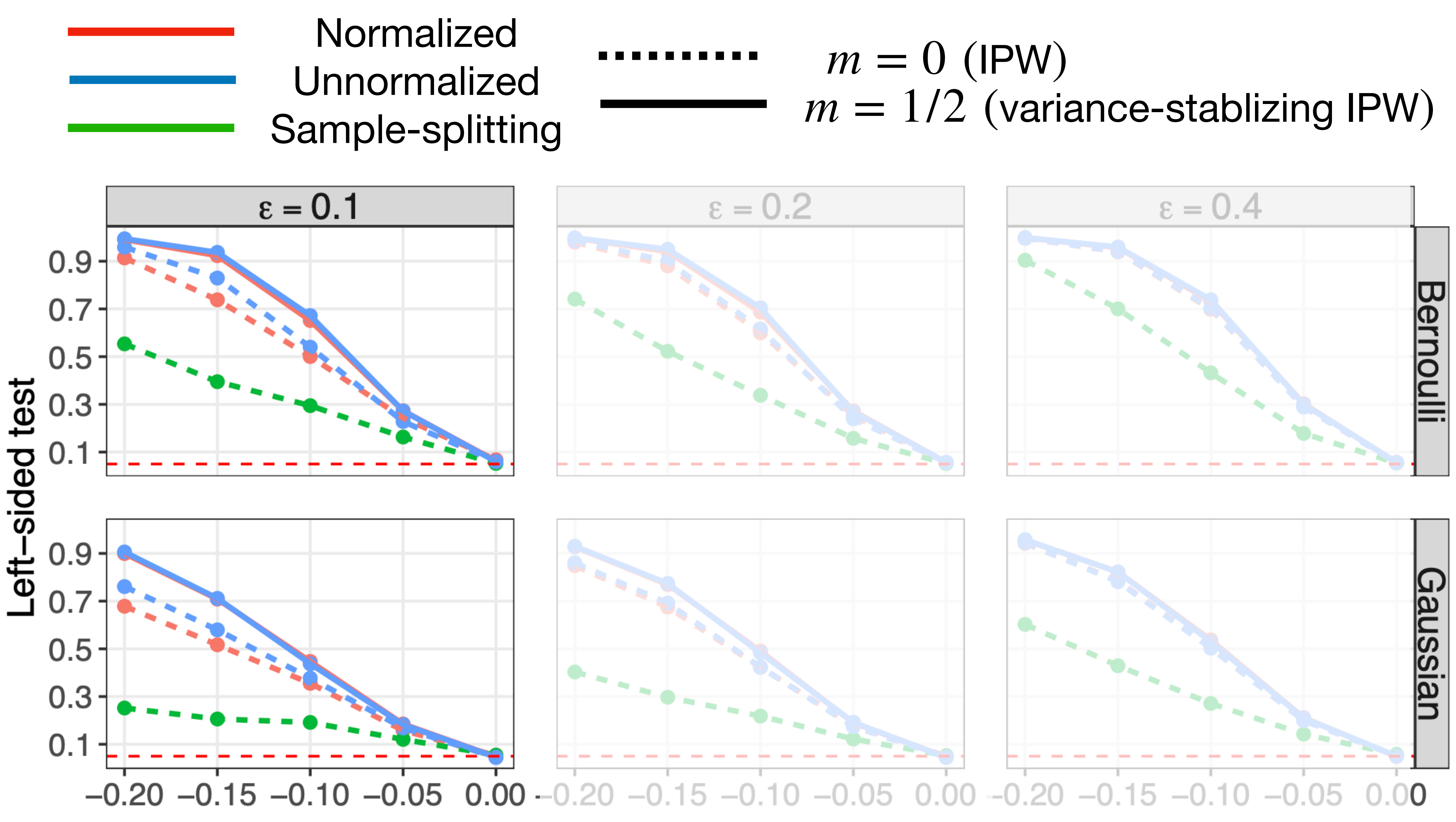

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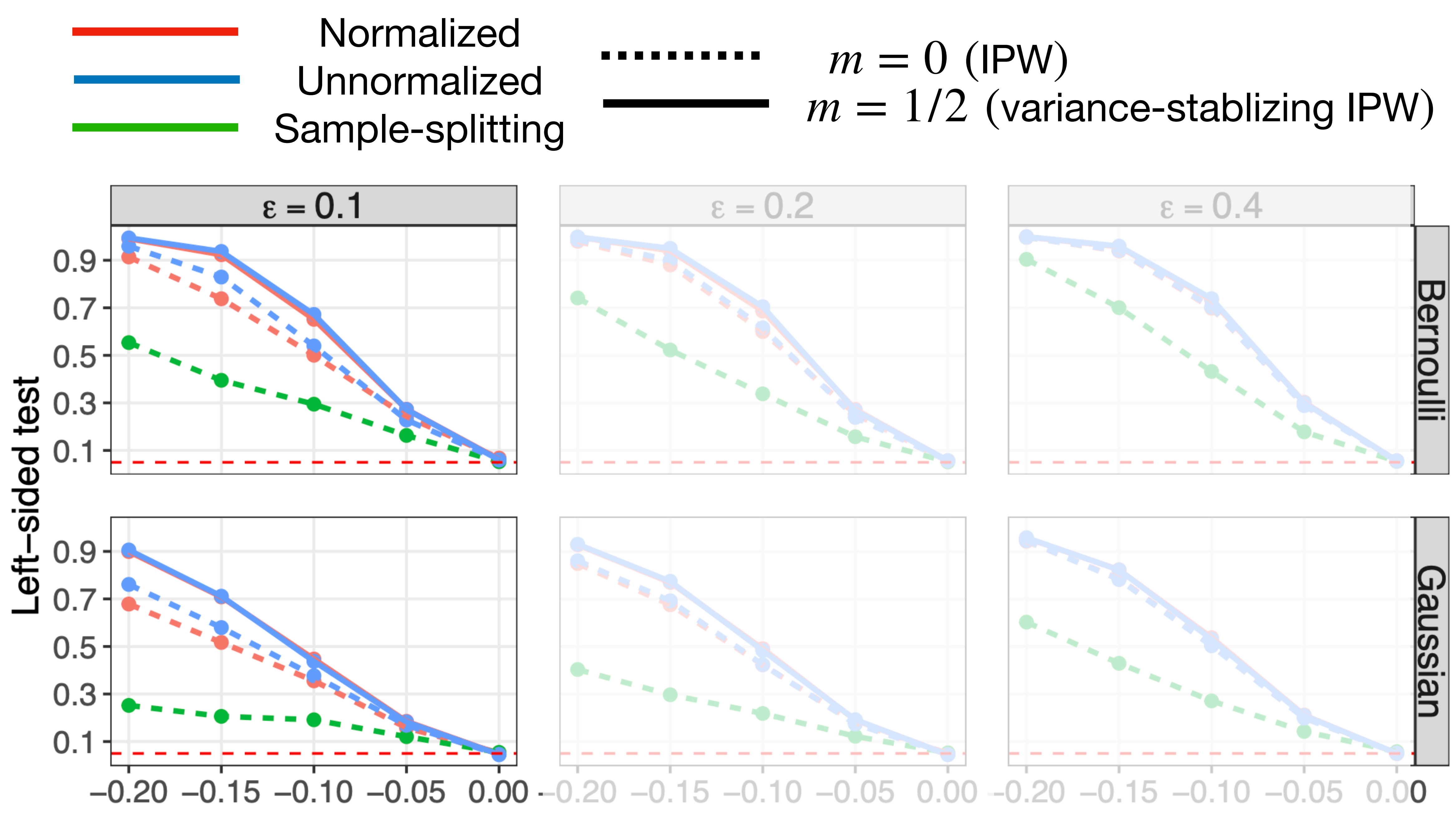

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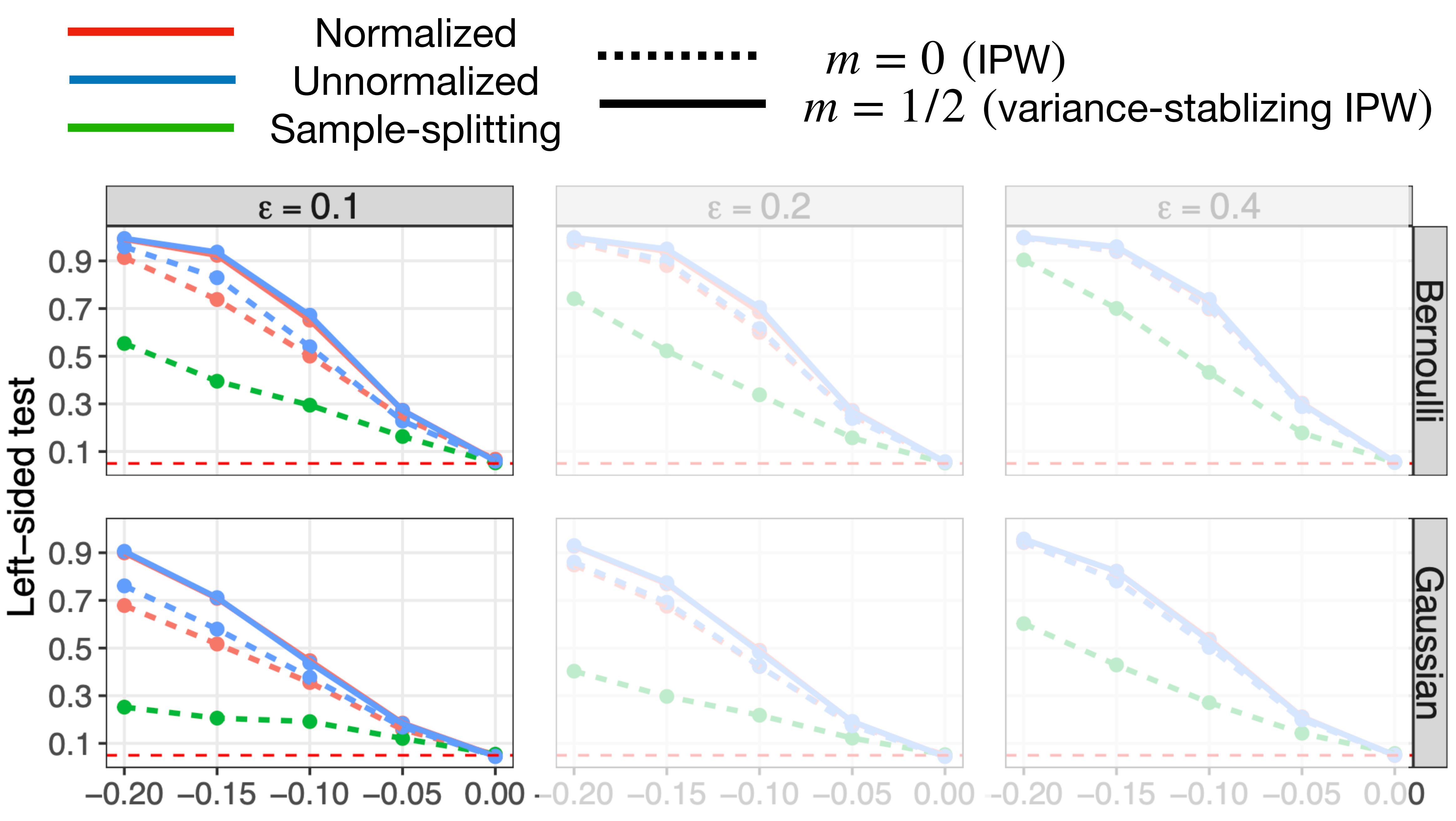

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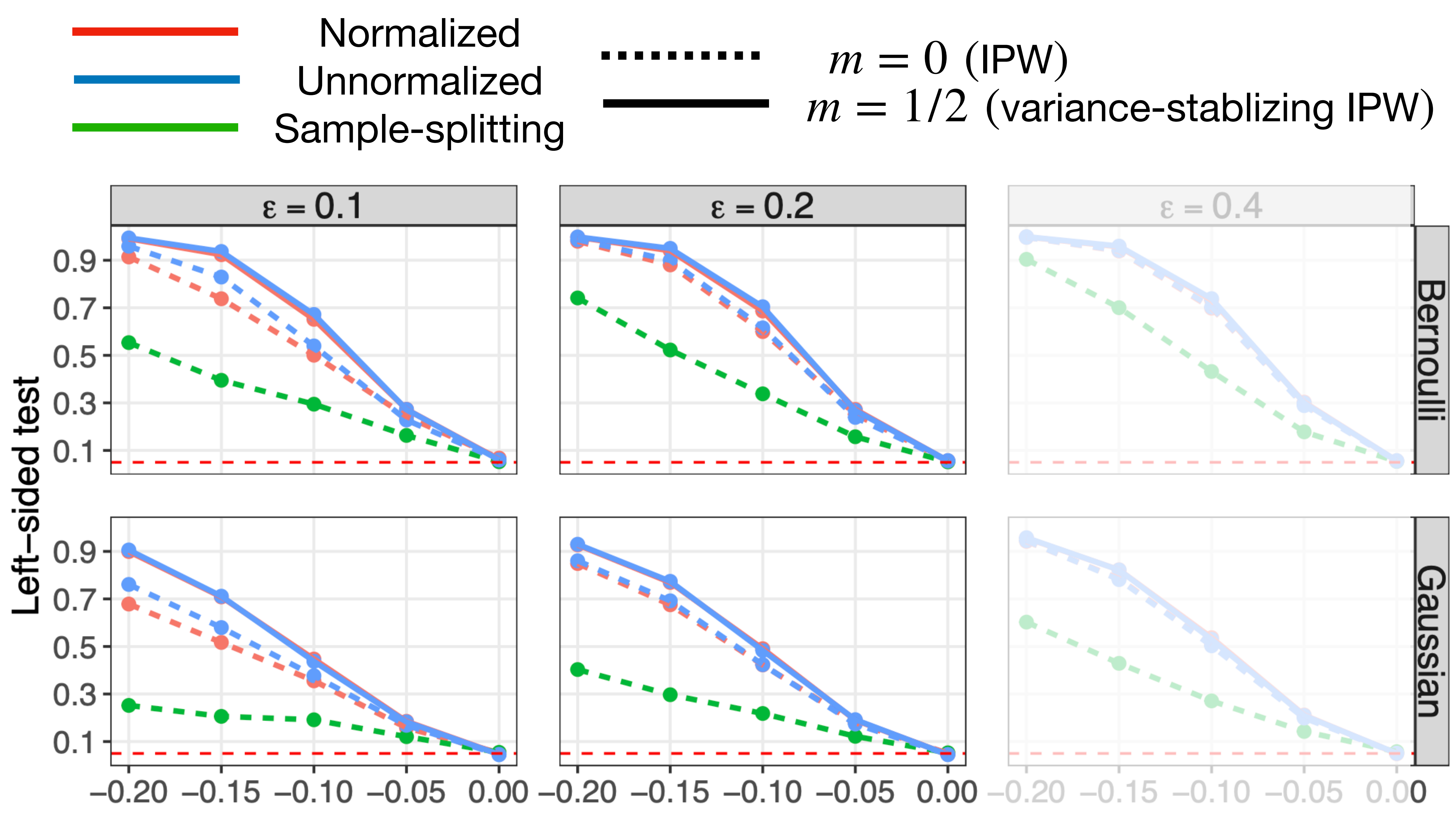

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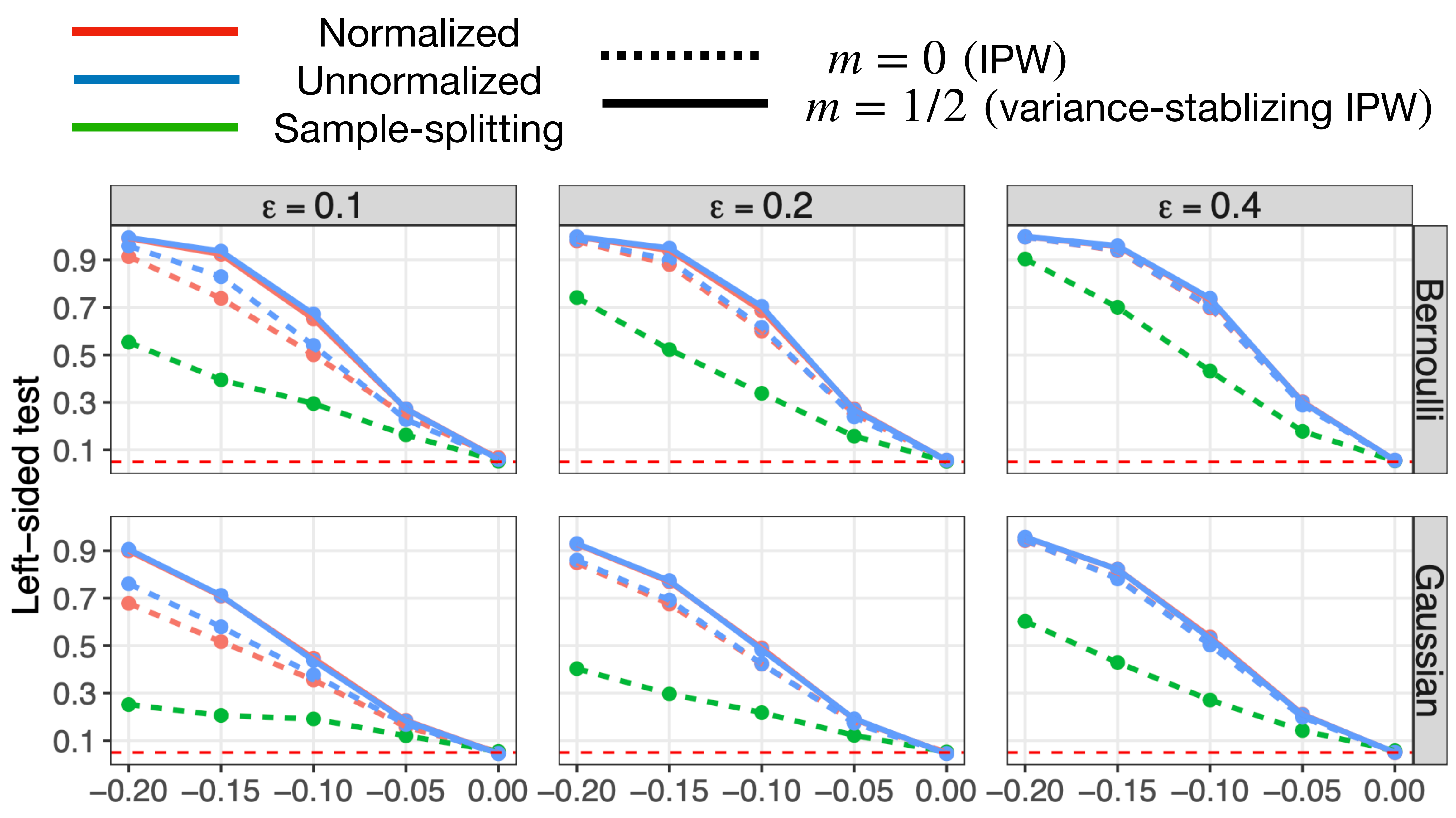

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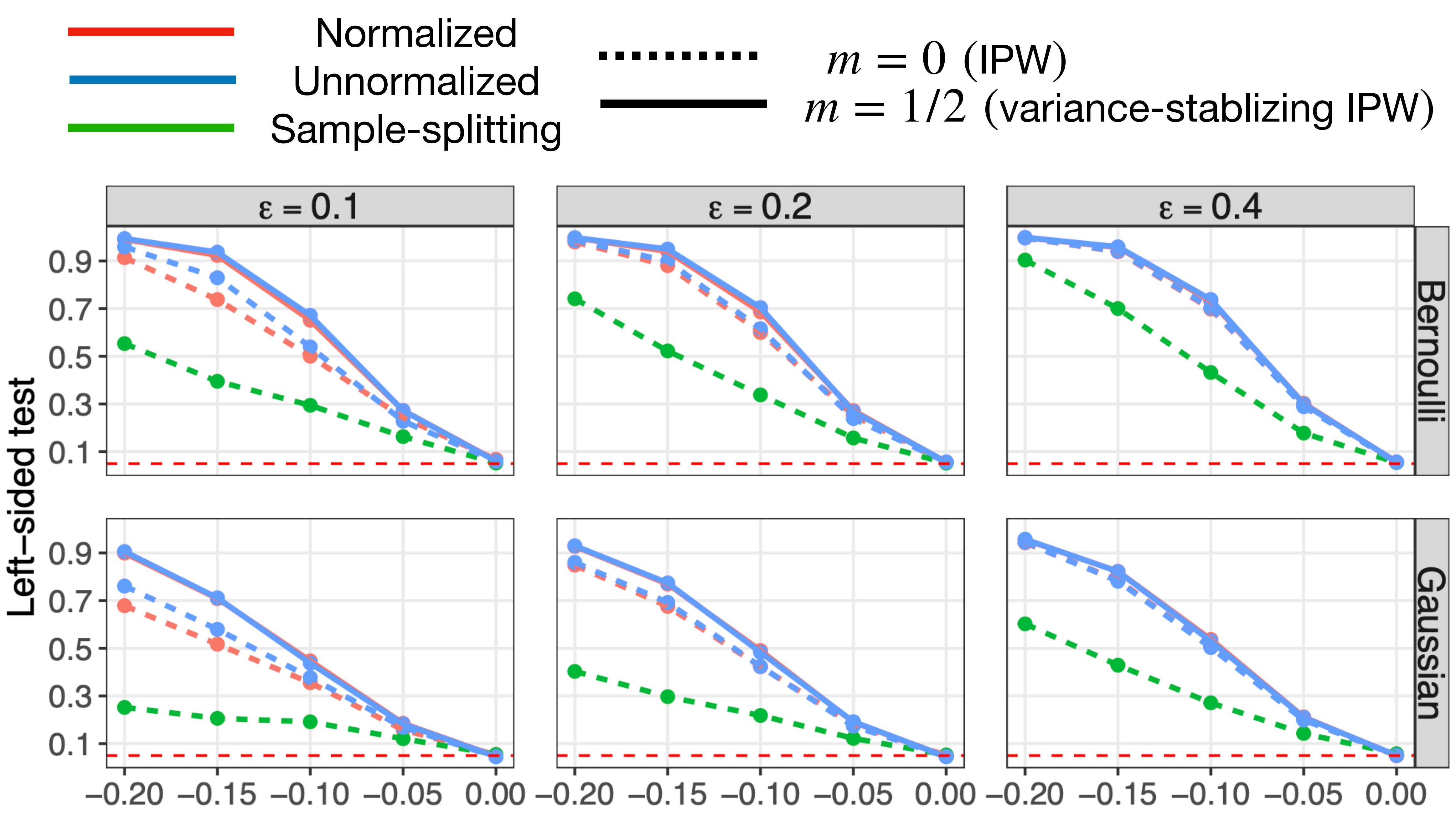

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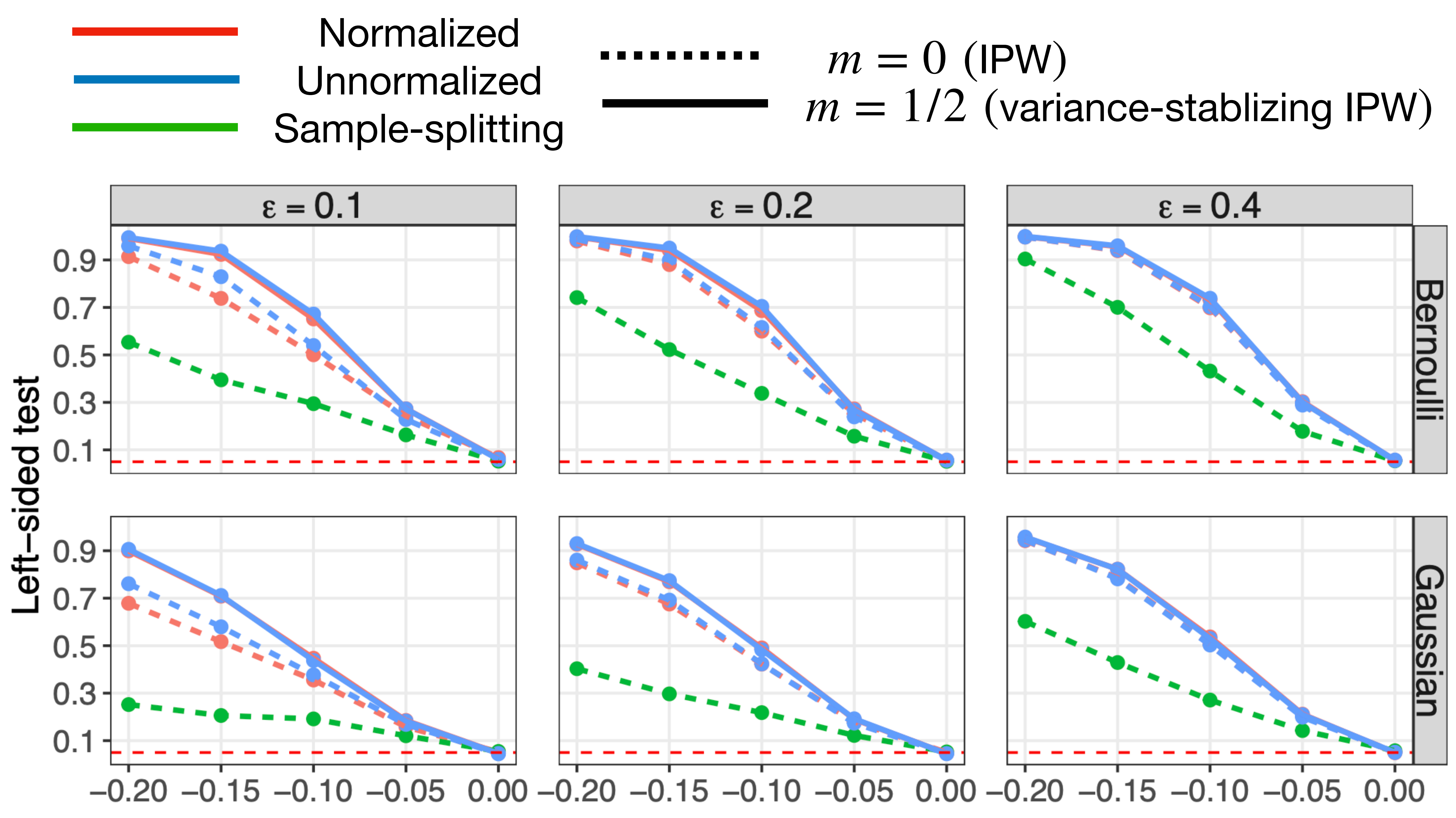

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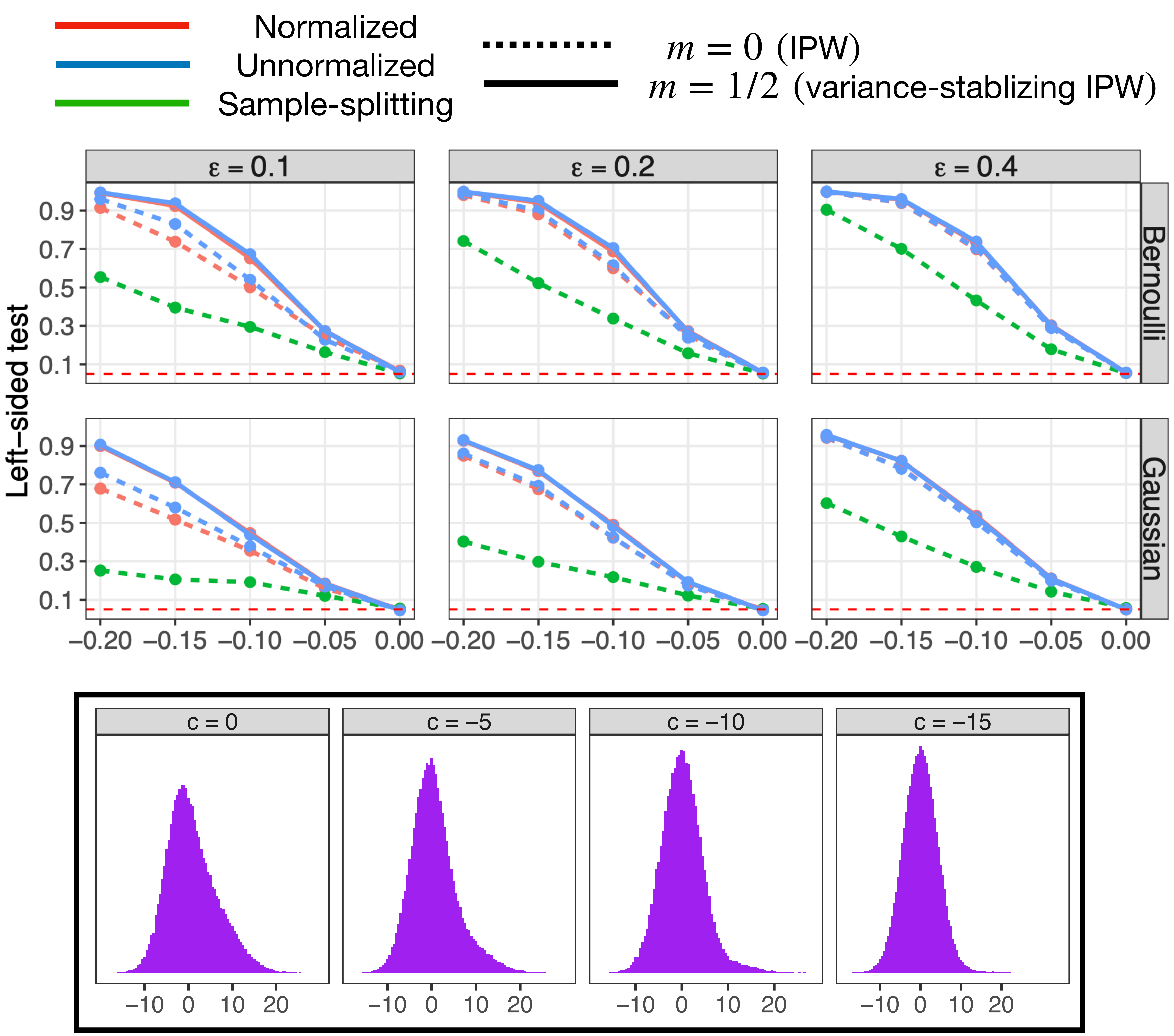

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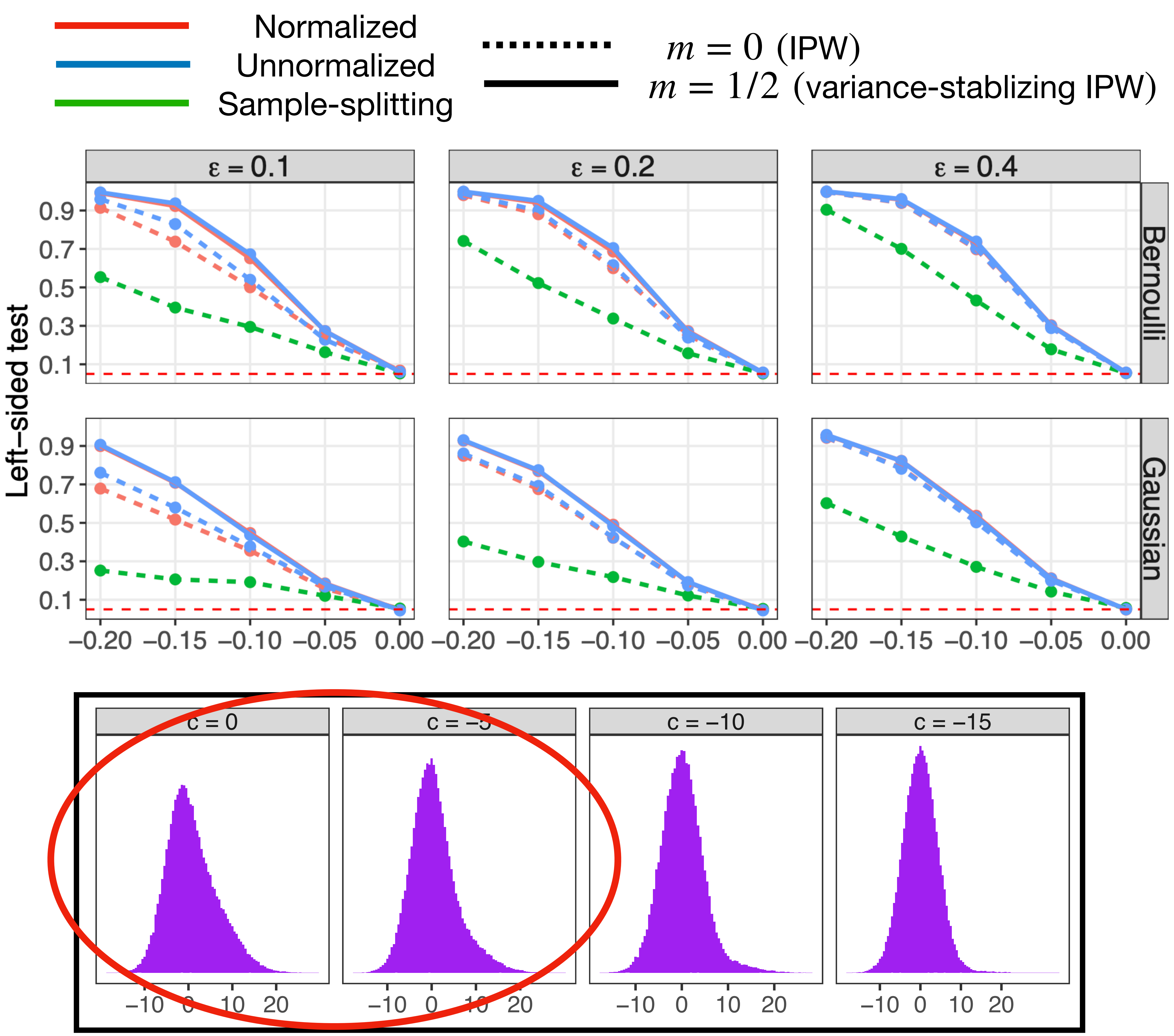

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- Explore the optimality within the class of WAIPW test statistics.

Reference

Assumption-lean weak limits and tests for two-stage adaptive experiments. In arXiv, 2025.